MATHEMATICS

Student Textbook

Grade 8

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FEDERAL DEMOCRATIC REPUBLIC OF ETHIOPIA
MINISTRY OF EDUCATION

AL GHURAIR
PRINTING AND PUBLISHING LLC

Acknowledgements

The redesign, printing and distribution of this student textbook has been funded through the General Education Quality Improvement Project (GEQIP), which aims to improve the quality of education for Grades 1–12 students in government schools throughout Ethiopia.

The Federal Democratic Republic of Ethiopia received funding for GEQIP through credit/financing from the International Development Associations (IDA), the Fast Track Initiative Catalytic Fund (FTI CF) and other development partners – Finland, Italian Development Cooperation, the Netherlands and UK aid from the Department for International Development (DFID).

The Ministry of Education wishes to thank the many individuals, groups and other bodies involved – directly and indirectly – in publishing the textbook and accompanying teacher guide.

Every effort has been made to trace the copyright holders of the images and we apologise in advance for any unintentional omission. We would be pleased to insert the appropriate acknowledgement in any subsequent edition of this publication.

First edition, 2003(E.C.)

ISBN: 978-99944-2-056-8

Developed, printed and distributed for the Federal Democratic Republic of Ethiopia, Ministry of Education by:

Al Ghurair Printing and Publishing House CO. (LLC)
PO Box 5613
Dubai
U.A.E.

In collaboration with

Kuraz International Publisher P.L.C
P.O. Box 100767
Addis Ababa
Ethiopia

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UNIT 7: GEOMETRY AND MEASUREMENTS

UNIT 1

SQUARES, SQUARE ROOTS, CUBES AND CUBE ROOTS

Unit outcomes

After completing this unit, you should be able to:

- understand the notion square and square roots and cubes and cube roots.
- determine the square roots of the perfect square numbers.
- extract the approximate square roots of numbers by using the numerical table.
- determine cubes of numbers.
- extract the cube roots of perfect cubes.

Introduction

What you had learnt in the previous grade about multiplication will be used in this unit to describe special products known as squares and cubes of a given numbers. You will also learn what is meant by square roots and cube roots and how to compute them. What you will learn in this unit are basic and very important concepts in mathematics. So get ready and be attentive!

1.1 The Square of a Number

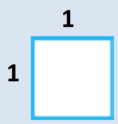
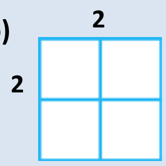
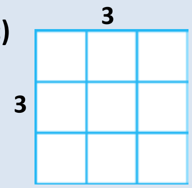
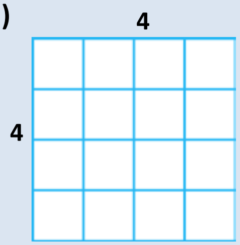
1.1.1 Square of a Rational Number

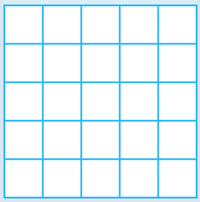
Addition and subtraction are operations of the first kind while multiplication and division are operation of the second kind. Operations of the third kind are **raising to a power** and **extracting roots**. In this unit, you will learn about raising a given number to the power of "2" and power of "3" and extracting square roots and cube roots of some perfect squares and cubes.

Group Work 1.1

Discuss with your friends

1. Complete this Table 1.1. Number of small squares

	Standard Form	Factor Form	Power Form
a) 	1	1×1	1^2
b) 	4	2×2	2^2
c) 	________	________	________
d) 	________	________	________

e)  **5** **5** **Figure 1.1**	____________	____________	____________

2. Put three different numbers in the circles so that when you add the numbers at the end of each – line you always get a square number.

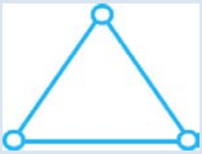


Figure 1.2

3. Put four different numbers in the circles so that when you add the numbers at the end of each line you always get a square number.

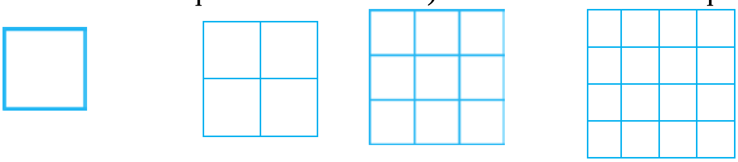
Figure 1.3

Definition 1.1: **The process of multiplying a rational number by itself is called squaring the number.**

For example some few square numbers are:

a) $1 \times 1 = 1$ is the 1^{st} square number. c) $3\times 3 = 9$ is the 3^{rd} square number.

b) $2\times 2 = 4$ is the 2^{nd} square number. d) $4\times4 = 16$ is the 4^{th} square number.



a) **b)** **c)** **d)**

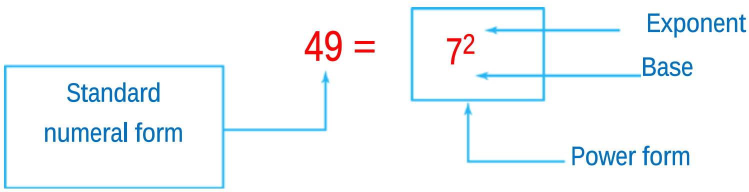
Figure 1.4 A square number can be shown as a pattern of squares

If the number to be multiplied by itself is ‘a’, then the product (or the result $a \times a$) is usually written as a^2 and is read as:

- ✓ a squared or
- ✓ the square of a or
- ✓ a to the power of 2

In geometry, for example you have studied that the area of a square of side length ‘a’ is $a \times a$ or briefly a^2.

When the same number is used as a factor for several times, you can use an exponent to show how many times this numbers is taken as a factor or base.



Note: 7^2 is read as

- ✓ 7 squared or
- ✓ the square of 7 or
- ✓ 7 to the power of 2

Example 1: Find the square of each of the following.

a) 8 b) 10 c) 14 d) 19

Solution

a) $8^2 = 8 \times 8 = 64$

b) $10^2 = 10 \times 10 = 100$

c) $14^2 = 14 \times 14 = 196$

d) $19^2 = 19 \times 19 = 361$

Example2: Identify the base, exponent, power form and standard form of the following expression.

a) 10^2 b) 18^2

Solution

a)

Standard numeral form

$100 = \boxed{10^2}$ exponent base

Power form

b)

Standard numeral form

$324 = \boxed{18^2}$ exponent base

Power form

Note: There is a difference between a^2 and $2a$. To see this distinction consider the following examples of comparison.

Example3: a) $30^2 = 30 \times 30 = 900$ while $2 \times 30 = 60$

b) $40^2 = 40 \times 40 = 1600$ while $2 \times 40 = 80$

c) $52^2 = 52 \times 52 = 2704$ while $2 \times 52 = 104$

Hence from the above example; you can generalize that $a^2 = a \times a$ and $2a = a + a$, are quite different expressions.

Definition 1.2: A rational number x is called a perfect square, if and only if $x = n^2$ for some $n \in \mathbb{Q}$.

Example 4: $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$. Thus 1, 4, 9, 16 and 25 are perfect squares.

Note: A perfect square is a number that is a product of a rational number times itself and its square root is a rational number.

Example5: In Table 1.2 below some natural numbers are given as values of x. Find x^2 and complete table 1.2.

x	1	2	3	4	5	10	15	20	25	35
x^2										

Solution

When $x = 1$, $x^2 = 1^2 = 1 \times 1 = 1$

When $x = 2$, $x^2 = 2^2 = 2 \times 2 = 4$

When $x = 3$, $x^2 = 3^2 = 3 \times 3 = 9$

When $x = 4$, $x^2 = 4^2 = 4 \times 4 = 16$

When $x = 5$, $x^2 = 5^2 = 5 \times 5 = 25$

When $x = 10$, $x^2 = 10^2 = 10 \times 10 = 100$

When $x = 15$, $x^2 = 15^2 = 15 \times 15 = 225$

When $x = 20$, $x^2 = 20^2 = 20 \times 20 = 400$

When $x = 25$, $x^2 = 25^2 = 25 \times 25 = 625$

When $x = 35$, $x^2 = 35^2 = 35 \times 35 = 1225$

x	1	2	3	4	5	10	15	20	25	35
x^2	1	4	9	16	25	100	225	400	625	1225

You have so far been able to recognize the squares of natural numbers, you also know that multiplication is closed in the set of rational numbers. Hence it is possible to multiply any rational number by itself.

Example 6: Find x^2 in each of the following where x is rational number given as:

a) $x = \frac{4}{3}$ b) $x = \frac{1}{3}$ c) $x = \frac{3}{5}$ d) $x = 0.26$

Solution

a) $x^2 = \left(\frac{4}{3}\right)^2 = \frac{4}{3} \times \frac{4}{3} = \frac{4 \times 4}{3 \times 3} = \frac{16}{9}$

b) $x^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{3} \times \frac{1}{3} = \frac{1 \times 1}{3 \times 3} = \frac{1}{9}$

c) $x^2 = \left(\frac{3}{5}\right)^2 = \frac{3}{5} \times \frac{3}{5} = \frac{3 \times 3}{5 \times 5} = \frac{9}{25}$

d) $x^2 = (0.26)^2 = \left(\frac{26}{100}\right)^2 = \frac{26}{100} \times \frac{26}{100} = \frac{26 \times 26}{100 \times 100} = \frac{676}{10,000}$

Note:

i. The squares of natural numbers are also natural numbers.

ii. $0 \times 0 = 0$ therefore $0^2 = 0$

iii. We give no meaning to the symbol 0^0

iv. If $a \in \mathbb{Q}$ and $a \neq 0$, then $a^0 = 1$

v. For any rational number ‘a’, $a \times a$ is denoted by a^2 and read as “a squared” or “a to the power of 2” or “the square of a”.

Exercise 1A

1. Determine whether each of the following statements is true or false.

a) $15^2 = 15 \times 15$ d) $81^2 = 2 \times 81$ g) $x^2 = 2^x$

b) $20^2 = 20 \times 20$ e) $41 \times 41 = 41^2$ h) $x^2 = 2^{2x}$

c) $19^2 = 19 \times 19$ f) $-(50)^2 = 2500$ i) $(-60)^2 = 3600$

2. Complete the following.

a) $12 \times$ _____ $= 144$ d) $(3a)^2 =$ ___ $\times$ ___

b) $51 \times$ _____ $= 2601$ e) $8a =$ ___ $+$ ____

c) $60^2 =$ _____ $\times$ _____ f) $28 \times 28 =$ ______

3. Find the square of each of the following.

a) 8 b) 12 c) 19 d) 51 e) 63 f) 100

4. Find x^2 in each of the following.

a) $x = 6$ c) $x = -0.3$ e) $x = \frac{-50}{3}$ g) $x = 0.07$

b) $x = \frac{1}{6}$ d) $x = -20$ f) $x = 56$

5. a. write down a table of square numbers from the first to the tenth.

b. Find two square numbers which add to give a square number.

6. Explain whether:

a. 441 is a square number. c. 1007 is a square number.

b. 2001 is a square number.

Challenge Problems

7. Find
   a) The 8^{th} square number.
   b) The 12^{th} square number.
   c) The first 12 square numbers.
8. From the list given below indicate all numbers that are perfect squares.
   a) 50 20 64 30 1 80 8 49 9
   b) 10 21 57 4 60 125 7 27 48 16 25 90
   c) 137 150 75 110 50 625 64 81 144
   d) 90 180 216 100 81 75 140 169 125
9. Show that the difference between any two consecutive square numbers is an odd number.
10. Show that the difference between the 7^{th} square number and the 4^{th} square number is a multiple of 3.

Theorem1.1: Existence theorem
For each rational number x, there is a rational number y ($y \geq 0$) such that $x^2 = y$.

Example 7: By the existence theorem, if

a) $x = 9$, then $y = 9^2 = 81$

b) $x = 0.5$, then $y = (0.5)^2 = 0.25$

c) $x = -17$, then $y = (-17)^2 = 289$

d) $x = \frac{7}{11}$, then $y = \left(\frac{7}{11}\right)^2 = \frac{49}{121}$

Rough calculation could be carried out for approximating and checking the results in squaring rational numbers. Such an approximation depends on rounding off decimal numbers as it will be seen from the following examples.

Examples 8: Find the approximate values of x^2 in each of the following:

a) $x = 3.4$

b) $x = 9.7$

c) $x = 0.026$

Solution

a) $3.4 \approx 3$ thus $(3.4)^2 \approx 3^2 = 9$

b) $9.7 \approx 10$ thus $(9.7)^2 \approx 10^2 = 100$

c) $0.026 \approx 0.03$ thus $(0.026)^2 \approx \left(\frac{3}{100}\right)^2 = 0.0009$

Exercise 1B

1. Determine whether each of the following statements is true or false.
   a) $(4.2)^2$ is between 16 and 25
   b) $0^2 = 2$
   c) $11^2 > (11.012)^2 > 12^2$
   d) $(9.9)^2 = 100$
   e) $(-13)^2 = -169$
   f) $81 \times 27 = 9^2 \times 9\times3$
2. Find the approximate values of x^2 in each of the following.
   a) $x = 3.2$
   b) $x = 9.8$
   c) $x = -12.1$
   d) $x = 2.95$
   e) $x = 0.086$
   f) $x = 8.80$
3. Find the square of the following numbers and check your answers by rough calculation.
   a) 0.87
   b) 16.45
   c) 12.12
   d) 42. 05
   e) 25. 14
   f) 28. 23
   g) 38.9
   h) 54. 88

1.1.2 Use of Table of Values of Squares

Activity 1.1

Discuss with your friends / partners/

Use table of square to find x^2 in each of the following.

a) $x = 1.08$
b) $x = 2.26$
c) $x = 9.99$
d) $x = 1.56$
e) $x = 5.48$
f) $x = 7.56$

- ✓ To find the square of a rational number when it is written in the form of a decimal is tedious and time consuming work. To avoid this tedious and time consuming work a table of squares is prepared and presented in the "Numerical tables" at the end of this book.
- ✓ In this table the first column headed by x lists numbers starting from 1.0. The remaining columns are headed respectively by the digits 0 to 9.

Now if you want to determine the square of a number for example 2.54 proceed as follows.

Step i. Under the column headed by x, find the row with 2.5.

Step ii. Move to the right along the row until you get the column under 4, (or find the column headed by 4).

Step iii. Then read the number at the intersection of the row in (i) and the column (ii), (see the illustration below).

Hence $(2.54)^2 = 6.452$

X	0	1	2	3	4	5	6	7	8	9
1,0										
⋮										
2,0										
2.5					6.452					
3,0										
⋮										
4,0										
⋮										
5,0										
⋮										
6,0										
⋮										
7,0										
⋮										
8.0										
9.0										

Figure 1.5 Tables of squares

Note that the steps (i) to (iii) are often shortened by saying "2.5 under 4".

- ✓ Mostly the values obtained from the table of squares are only approximate values which of course serves almost for all practical purposes.

Group work 1.2

Discuss with your group.

Find the square of the number 8.95

a) use rough calculation method.

b) use the numerical table.

c) by calculating the exact value of the number.
d) compare your answer from "a" to "c".
e) write your generalization.

Example9: Find the square of the number 4.95.

Solution: Do rough calculation and compare your answer with the value obtained from the table.

i. Rough calculation

$4.95 \approx 5$ and $5^2 = 25$

$(4.95)^2 \approx 25$

ii. Value obtained from the table

i) Find the row which starts with 4.9.

ii) Find the column headed by 5.

iii) Read the number, that is $(4.95)^2$ at the intersection of the row in (i) and the column in (ii) ; $(4.95)^2 = 24.50$.

iii. Exact Value

Multiply 4.95 by 4.95

$4.95 \times 4.95 = 24.5025$

Therefore $(4.95)^2 = 24.5025$.

This example shows that the result obtained from the "Numerical table" is an approximation and more closer to the exact value.

Exercise 1C

1. Determine whether each of the following statements is true or false.

a) $(2.3)^2 = 5.429$ c) $(3.56)^2 = 30.91$ e) $(5.67)^2 = 32$

b) $(9.1)^2 = 973.2$ d) $(9.90)^2 = 98.01$ f) $(4.36)^2 = 16.2$

2. Find the squares of the following numbers from the table.

a) 4.85 c) 88.2 e) 2.60 g) 498 i) 165

b) 6.46 d) 29.0 f) $\frac{3}{2}$ h) 246

1.2 The Square Root of a Rational Number

Group Work 1.3

Discuss with your Friends

Find the square root of each of the following numbers.

a) 81 **b)** 324 **c)** $\frac{1}{4}$ **d)** $\frac{64}{49}$ **e)** $\frac{4}{49}$ **f)** $\frac{25}{625}$

✓ So far you have studied the meaning of x^2 when x is a rational number. It is now logical to ask, whether you can go in the reverse (or in the opposite direction) or not. In this sub unit you will answer this and related questions in a more systematized manner.

Definition 1.3: Square roots

For any two rational numbers a and b if $a^2 = b$, then a is called the square root of b.

Example 10:

a) 4 is the square root of 16, since $4^2 = 16$.

b) 5 is the square root of 25, since $5^2 = 25$.

c) 6 is the square root of 36, since $6^2 = 36$.

Example 11: The area of a square is $49m^2$. What is the length of each side?

Solution:

$$\ell \times w = A$$

$$s \times s = 49\ m^2$$

$$s^2 = 49\ m^2$$

$$s = 7m$$

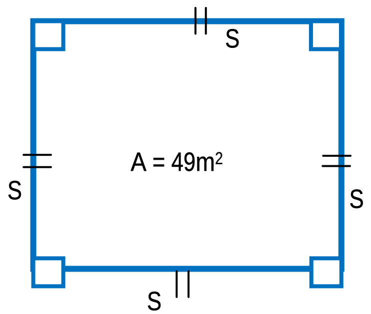


Figure 1.6

The length of each side is 7 meters. This is one way to express the mathematical relationship "7 is the square root of 49" because $7^2 = 49$.

Note:

i. The notion "*square root*" is the inverse of the notion "*square of a number*".

ii. The operation "*extracting square root*" is the inverse of the operation "*squaring*".

iii. In extracting square roots of rational numbers, first decompose the number into a product consisting of two equal factors and take one of the equal factors as the square root of the given number.

iv. The symbol or notation for square root is "$\sqrt{\ }$" it is called radical sign.

v. For $b \geq 0$, the expression $\sqrt{b}$ is called radical b and the number b is called a radical.

vi. The relation of squaring and square root can be expressed as follows:

$$a \xrightarrow{\text{Squaring}} a^2 = b, \text{ for } b \geq 0$$

$$a \xleftarrow{\text{Square root}} a^2$$

vii. a is the square root of b and written as $a = \sqrt{b}$.

Example 12: Find the square root of x, if x is:

a) 100 b) 125 c) 169 d) 256 e) 625 f) 1600

Solution

a) $x = 100 = 10 \times 10$

$x = 10^2$, thus the square root of 100 is 10.

b) $x = 225 = 15 \times 15$

$x = 15^2$, thus the square root of 225 is 15.

c) $x = 169 = 13 \times 13$

$x = 13^2$, thus the square root of 169 is 13.

d) $x = 256 = 16 \times 16$

$x = 16^2$, thus the square root of 256 is 16.

e) $x = 625 = 25 \times 25$

$x = 25^2$, thus the square root of 625 is 25.

f) $x = 1600 = 40 \times 40$

$x = 40^2$, thus the square root of 1600 is 40.

Exercise 1D

1. Determine whether each of the following statements is true or false.

a) $\sqrt{0} = 0$

b) $\sqrt{25} = \pm 5$

c) $\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

d) $-\sqrt{121} = -11$

e) $-\sqrt{\frac{36}{324}} = \frac{1}{3}$

f) $\sqrt{\frac{324}{625}} = \frac{18}{25}$

g) $-\sqrt{\frac{900}{961}} = -\frac{30}{31}$

2. Find the square root of each of the following numbers.

a) 121 b) 144 c) 289 d) 361 e) 400 f) 441 g) 484 h) 529

3. Evaluate each of the following.

a) $\sqrt{\frac{1}{25}}$

b) $\sqrt{\frac{1}{81}}$

c) $-\sqrt{\frac{36}{144}}$

d) $-\sqrt{576}$

e) $\frac{\sqrt{529}}{\sqrt{625}}$

f) $-\sqrt{676}$

g) $\sqrt{729}$

h) $-\sqrt{784}$

i) $\sqrt{\frac{16}{25}}$

Challenge Problems

4. If $\frac{x}{y} = -2$. Find $\sqrt{\frac{x^2}{y^2} + \frac{y^2}{x^2}}$

5. Simplify: $\sqrt{(81)^2} + \sqrt{(49)^2}$

6. If $x = 16$ and $y = 625$. Find $\left(2\sqrt{x+y}\right)^2$.

Definition 1.4 : If a number $y \geq 0$ is the square of a positive number x ($x \geq 0$), then the number x is called the square root of y.
This can be written as $x = \sqrt{y}$.

Example13: Find

a) $\sqrt{0.01}$ c) $\sqrt{0.81}$ e) $\sqrt{0.7921}$ g) $\sqrt{48.8601}$

b) $\sqrt{0.25}$ d) $\sqrt{0.6889}$ f) $\sqrt{0.9025}$

Solution

a) $\sqrt{0.01} = \sqrt{0.1 \times 0.1} = 0.1$

b) $\sqrt{0.25} = \sqrt{0.5 \times 0.5} = 0.5$

c) $\sqrt{0.81} = \sqrt{0.9 \times 0.9} = 0.9$

d) $\sqrt{0.6889} = \sqrt{0.83 \times 0.83} = 0.83$

e) $\sqrt{0.7921} = \sqrt{0.89 \times 0.89} = 0.89$

f) $\sqrt{0.9025} = \sqrt{0.95 \times 0.95} = 0.95$

g) $\sqrt{48.8601} = \sqrt{6.99 \times 6.99} = 6.99$

Exercise 1E

Simplify the square roots.

a) $\sqrt{35.88}$ c) $\sqrt{89.87}$ e) $\sqrt{62.25}$

b) $\sqrt{36.46}$ d) $\sqrt{99.80}$ f) $\sqrt{97.81}$

1.2.1 Square Roots of Perfect Squares

Group work 1.4

Discuss with your group.

1. **Find the prime factorization of the following numbers by using the factor trees.**

   a) 64 c) 121 e) 324 g) 625 i) 700

   b) 81 d) 289 f) 400 h) 676

Note: The following properties of squares are important:

$(ab)^2 = a^2 \times b^2$ and $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$ (where $b \neq 0$).

Thus $(2\times3)^2 = 2^2 \times 3^2 = 36$ and $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$.

Remember a number is called a perfect square, if it is the square of a rational number.

The following properties are useful to simplify square roots of numbers.

Properties of Square roots, for $a \geq 0$, $b \geq 0$.

If $\sqrt{a}$ and $\sqrt{b}$ represent rational numbers, then

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \text{ and } \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ where } b \neq 0.$$

Example: 14 Determine whether each of the following numbers is a perfect square or not.

a) 36 c) 81 e) $\frac{16}{625}$ g) 11

b) 49 d) $\frac{49}{25}$ f) 7

Solution: a) 36 is a perfect square, because $36 = 6^2$.

b) 49 is a perfect square, because $49 = 7^2$.

c) 81 is a perfect square, because $81 = 9^2$.

d) $\frac{49}{25}$ is a perfect square, becaus $\frac{49}{25} = \left(\frac{7}{5}\right)^2$.

e) $\frac{16}{625}$ is a perfect square, because $\frac{16}{625} = \left(\frac{4}{25}\right)^2$.

f) 7 is not a perfect square since there is no rational number whose square is equal to 7. In other words there is no rational number n such that $n^2 = 7$.

g) 11 is not a perfect square since there is no rational number whose square is equal to 11. In short there is no rational number n such that $n^2 = 11$.

Example 15: Use prime factorization and find the square root of each of the following numbers.

a) $\sqrt{324}$ b) $\sqrt{400}$ c) $\sqrt{484}$

Solution: a) $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$

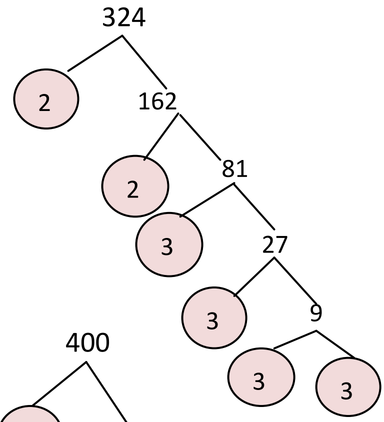
Now arrange the factors so that 324 is a product of two identical sets of prime factors.

i.e $324 = (2 \times 2 \times 3 \times 3 \times 3 \times 3)$

$= (2 \times 3 \times 3) \times (2 \times 3 \times 3)$

$= 18 \times 18 = 18^2$

So, $\sqrt{324} = \sqrt{18 \times 18} = 18$



b) $400 = (2 \times 2 \times 5) \times (2 \times 2 \times 5)$

Now arrange the factors so that 400 is a product of two identical sets of prime factors.

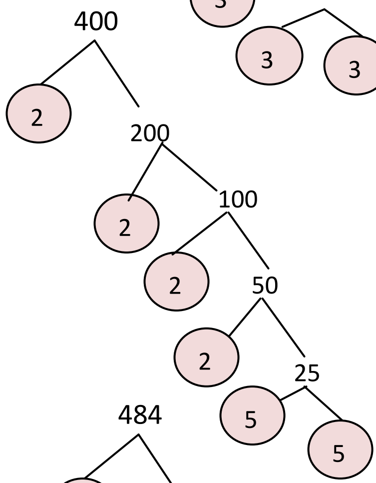
i.e $400 = (2 \times 2 \times 5) \times (2 \times 2 \times 5)$

$= 20 \times 20$

$= 20^2$

So $\sqrt{400} = \sqrt{20 \times 20}$

$= 20$



c) $484 = 2 \times 2 \times 11 \times 11$, now arrange the factors so that 484 is a product of two identical sets of prime factors.

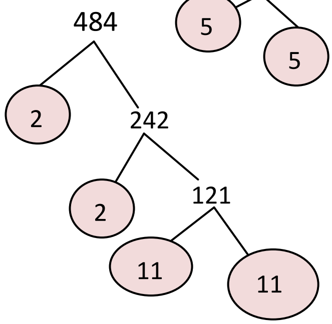
i.e $484 = 2 \times 2 \times 11 \times 11$

$= (2 \times 11) \times (2 \times 11)$

$= 22 \times 22 = 22^2$

So $\sqrt{484} = \sqrt{22 \times 22}$

$= 22$



Exercise 1F

1. Determine whether each of the following statements is true or false.

a) $\sqrt{64 \times 25} = \sqrt{64} \times \sqrt{25}$

b) $\sqrt{\dfrac{64}{4}} = 4$

c) $\sqrt{\dfrac{32}{64}} = \dfrac{\sqrt{32}}{\sqrt{64}}$

d) $\sqrt{\dfrac{0}{1296}} = 0$

e) $\sqrt{\dfrac{1296}{0}} = 1$

f) $\sqrt{\dfrac{729}{1444}} = \dfrac{27}{38}$

2. Evaluate each of the following.

a) $\sqrt{0.25}$ c) $\sqrt{\dfrac{1296}{1024}}$ e) $\sqrt{\dfrac{81}{324}}$

b) $\sqrt{0.0625}$ d) $\sqrt{\dfrac{625}{1024}}$ f) $\sqrt{\dfrac{144}{400}}$

Challenge Problem

3. Simplify a) $\sqrt{625-0} - \sqrt{172-3}$

b) $\sqrt{81 \times 625}$

c) $\sqrt{\left(\dfrac{1}{64}\right)^2}$

4. Does every number have two square roots? Explain.
5. Which of the following are perfect squares?
   {0, 1, 4, 7, 12, 16, 25, 30, 36, 42, 49}
6. Which of the following are perfect squares?
   {50, 64, 72, 81, 95, 100, 121, 140, 144, 169}
7. Copy and complete.
   a) $3^2 + 4^2 + 12^2 = 13^2$
   b) $5^2 + 6^2 +$ ____ = ______
   c) $6^2 + 7^2 +$ _______ = ______
   d) $x^2 + (x + 1)^2 +$ ____ = ____

Using the square root table

The same table which you can use to determine squares of numbers can be used to find the approximate square roots, of numbers.

Example 16: Find $\sqrt{17.89}$ from the numerical table.

Solution:

Step i. Find the number 17.89 in the body of the table for the function $y = x^2$.

Step ii. On the row containing this number move to the left and read 4.2 under x.

These are the first two digits of the square root of 17. 89

Step iii. To get the third digit start from 17.89 move vertically up ward and read 3.

Therefore $\sqrt{17.89} \approx 4.23$

X	0	1	2	3	4	5	6	7	8	9
1.0										
2.0										
				2^{nd}						
3.0										
4.0										
4.2		1^{st}		17.89						
5.0										
6.0										
7.0										
8.0										
9.9										

Figure 1.7 Table of square roots

If the radicand is not found in the body of the table, you can consider the number which is closer to it.

Example 17: Find $\sqrt{10.59}$

Solution: i) It is not possible to find the number 10. 59 directly in the table of squares. But in this case find two numbers in the table which are closer to it, one from left (i.e. 10.56) and one form right (10.63) that means $10.56 < 10.59 < 10.63$.

ii) Find the nearest number to (10.59) form those two numbers. So the nearest number is 10.56 thus $\sqrt{10.59} \approx \sqrt{10.56} = 3.25$.

Example 18: Find $\sqrt{83.60}$

Solution:

i. It is not possible to find the number 83. 60 directly in the table of squares. But find two numbers which are closer to it, one from left (i.e. 83.54) and one from right (i.e 83.72) that means $83.54 < 83.60 < 83.72$.

ii. Find the nearest number from these two numbers. Therefore the nearest number is 83. 54, so $\sqrt{83.60} \approx \sqrt{83.54} = 9.14$.

> **Note:** To find the square root of a number greater than 100 you can use the method illustrated by the following example.

Example 19: Find the square root of each of the following.

a) $\sqrt{6496}$ b) $\sqrt{9801}$ c) $\sqrt{9880}$ d) $\sqrt{9506}$

Solution:

a) $\sqrt{6496} = \sqrt{64.96\times100}$
$= \sqrt{64.96}\times\sqrt{100}$
$= 8.06\times10$
$= 80.6$

c) $\sqrt{9880} = \sqrt{98.80\times100}$
$= \sqrt{98.80}\times\sqrt{100}$
$= 9.94\times10$
$= 99.4$

b) $\sqrt{9801} = \sqrt{98.01\times100}$
$= \sqrt{98.01}\times\sqrt{100}$
$= 9.90\times10$
$= 99$

d) $\sqrt{9506} = \sqrt{95.06\times100}$
$= \sqrt{95.06}\times\sqrt{100}$
$= 9.75\times10$
$= 97.5$

Example 20: Find the square root of each of the following numbers by using the table

a) $\sqrt{98.41}$ c) $\sqrt{984100}$ e) $\sqrt{0.009841}$

b) $\sqrt{9841}$ d) $\sqrt{0.9841}$ f) $\sqrt{0.00009841}$

Solution:

a) $\sqrt{98.41} = 9.92$

b) $\sqrt{9841} = \sqrt{98.41 \times 100}$
$= \sqrt{98.41} \times \sqrt{100}$
$= 9.92 \times 10$
$= 99.2$

c) $\sqrt{984100} = \sqrt{98.41 \times 10000}$
$= \sqrt{98.41} \times \sqrt{10000}$
$= 9.92 \times 100$
$= 992$

d) $\sqrt{0.9841} = \sqrt{98.41 \times \frac{1}{100}}$
$= \sqrt{98.41} \times \sqrt{\frac{1}{100}}$
$= 9.92 \times \frac{1}{10}$
$= 0.992$

e) $\sqrt{0.009841} = \sqrt{98.41 \times \frac{1}{10{,}000}}$
$= \sqrt{98.41} \times \sqrt{\frac{1}{10{,}000}}$
$= 9.92 \times \frac{1}{100}$
$= 0.0992$

f) $\sqrt{0.00009841} = \sqrt{98.41} \times \sqrt{\frac{1}{1{,}000{,}000}}$
$= 9.92 \times \frac{1}{1{,}000}$
$= 0.00992$

Exercise 1G

1. Find the square root of each of the following numbers from the table.

a) 15.37	d) 153.1	g) 997	j) 5494
b) 40.70	e) 162.8	h) 6034	k) 5295
c) 121.3	f) 163.7	i) 6076	l) 3874

2. Use the table of squares to find approximate value of each of the following.

a) $\sqrt{6.553}$ c) $\sqrt{24.56}$

b) $\sqrt{8.761}$ d) $\sqrt{29.78}$

1.3 Cubes and Cube Roots

1.3.1 Cube of a Number

If the number to be cubed is ‘a’, then the product $a \times a \times a$ which is usually written as a^3 and is read as ‘a’ cubed. For example 3 cubed gives 27 because $3 \times 3 \times 3 = 27$.

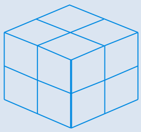
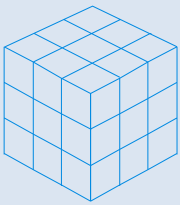
The product $3\times3\times3$ can be written as 3^3 and which is read as 3 cubed.

Activity 1.2

Discuss with your friends

1. Copy and complete this Table 1.3

   Number of small cubes

	Standard form	Factor form	Power form
a) 	1	$1 \times 1 \times 1$	1^3
b) 	8	$2 \times$ ___ $\times$ ___	2^3
c)  Figure 1.8	27	___ $\times$ ___ $\times$ ___	$\square^{\square}$

2. a) Which of these numbers are cubic numbers?

   64 100 125 216 500 1000
   1728 3150 4096 8000 8820 15625

   b) Write the cubic numbers from part (a) in power form.

3. Find a^3 in each of the following.

   a) $a = 2$ c) $a = 10$ e) $a = 0.5$

   b) $a = -2$ d) $a = \frac{1}{4}$ f) $a = 0.25$

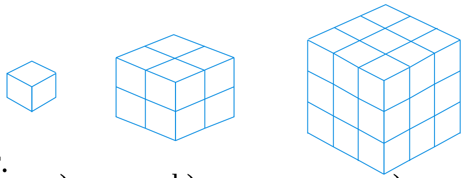
Definition 1.5: A cube number is the result of multiplying a rational number by itself, then multiplying by the number again.

For example, some few cube numbers are:

a) $1 \times 1 \times 1 = 1$ is the 1^{st} cube number.

b) $2 \times 2 \times 2 = 8$ is the 2^{nd} cube number.

c) $3 \times 3 \times 3 = 27$ is the 3^{rd} cube number.



a) b) c)

Figure 1.9 A cube number can be shown as a pattern of cubes

Example 21: Find the numbers whose cube are the following.

a) 4,913 b) 6,859 c) 9,261 d) 29,791

Solution:

a) $4,913 = 17 \times 17 \times 17 = 17^3$

b) $6,859 = 19 \times 19 \times 19 = 19^3$

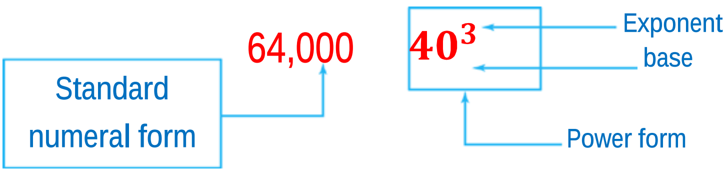
c) $9,261 = 21 \times 21 \times 21 = 21^3$

d) $29,791 = 31 \times 31 \times 31 = 31^3$

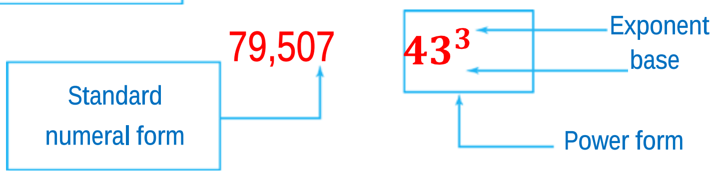
Example 22: Identify the base, exponent, power form and standard numeral form:

a) 40^3 b) 43^3

Solution: a)



b)



Example 23: In Table1.4 below integers are as values of x, find x^3 and complete the table 1.4.

x	-4	-3	-2	-1	0	1	2	3	4	5	6
x^3											

Solution:

When $x = -4$, $x^3 = (-4)^3 = -4 \times -4 \times -4 = -64$
When $x = -3$, $x^3 = (-3)^3 = -3 \times -3 \times -3 = -27$
When $x = -2$, $x^3 = (-2)^3 = -2 \times -2 \times -2 = -8$
When $x = -1$, $x^3 = (-1)^3 = -1 \times -1 \times -1 = -1$
When $x = 0$, $x^3 = 0^3 = 0 \times 0 \times 0 = 0$
When $x = 1$, $x^3 = 1^3 = 1 \times 1 \times 1 = 1$
When $x = 2$, $x^3 = 2^3 = 2 \times 2 \times 2 = 8$
When $x = 3$, $x^3 = 3^3 = 3 \times 3 \times 3 = 27$
When $x = 4$, $x^3 = 4^3 = 4 \times 4 \times 4 = 64$
When $x = 5$, $x^3 = 5^3 = 5 \times 5 \times 5 = 125$
When $x = 6$, $x^3 = 6^3 = 6 \times 6 \times 6 = 216$

Lastly you have:

x	-4	-3	-2	-1	0	1	2	3	4	5	6
x^3	-64	-27	-8	-1	0	1	8	27	64	125	216

The examples above illustrate the following theorem. This theorem is called **existence theorem.**

Theorem 1.2: Existence theorem
For each rational number x, there is a rational number y such that $y = x^3$.

Rough calculations could be used for approximating and checking the results in cubing rational numbers. The following examples illustrate the situation.

Example 24: Find the approximate values of x^3 in each of the following.

a) $x = 2.2$ b) $x = 0.065$ c) $x = 9.54$

Solution:

a. $2.2 \approx 2$ thus $(2.2)^3 \approx 2^3 = 8$

b. $0.065 \approx 0.07$ thus $(0.065)^3 \approx \left(\frac{7}{100}\right)^3 = \frac{343}{1{,}000{,}000}$
$= 0.000343$

c. $9.54 \approx 10$ thus $(9.54)^3 \approx 10^3 = 1{,}000$

Exercise 1H

1. Determine whether each of the following statements is true or false.

   a) $4^3 = 16 \times 4$ c) $(-3)^3 = 27$ e) $\left(\frac{4}{3}\right)^3 = \frac{64}{125}$

   b) $4^3 = 64$ d) $\left(\frac{3}{4}\right)^3 = \frac{27}{16}$ f) $\sqrt[3]{64} = 4$

2. Find x^3 in each of the following.

   a) $x = 8$ c) $x = -4$ e) $x = \frac{-1}{5}$

   b) $x = 0.4$ d) $x = -\frac{1}{4}$ f) $x = -0.2$

3. Find the approximate values of x^3 in each of the following.

   a) $x = -2.49$ c) $x = 2.98$

   b) $x = 2.29$ d) $x = 0.025$

Challenge Problem

4. The dimensions of a cuboid are 4xcm, 6xcm and 10xcm. Find
   a) The total surface area
   b) The volume

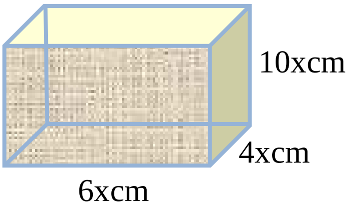


Figure 1.10 Cuboid

Table of Cubes

Activity 1.3

Discuss with your friends

Use the table of cubes to find the cubes of each of the following.

a) 2.26 c) 5.99 e) 8.86 g) 9.58 i) 9.99

b) 5.12 d) 8.48 f) 9.48 h) 9.89 j) 9.10

To find the cubes of a rational number when it is written in the form of a decimal is tedious and time consuming work. To avoid this tedious and time consuming work, a table of cubes is prepared and presented in the "Numerical Tables" at the end of this textbook.

In this table the first column headed by ‘x’ lists numbers starting from 1.0. The remaining columns are headed respectively by the digit 0 to 9.0. Now if we want to determine the cube of a number, for example 1.95 Proceed as follows.

Step i. Find the row which starts with 1.9 (or under the column headed by x).

Step ii. Move to the right until you get the number under column 5 (or find the column headed by 5).

Step iii. Then read the number at the intersection of the row in step (i) and the column step (ii) therefore we find that $(1.95)^3 = 7.415$. See the illustration below.

X	0	1	2	3	4	5	6	7	8	9
1.0										
1.9						7.415				
2.0										
3.0										
4.0										
5.0										
6.0										
7.0										
8.0										
9.0										

Figure 1.11 Tables of cubes

Note that the steps (i) to (iii) are often shortened saying “1.9 under 5”

Mostly the values obtained from the table of cubes are only a approximate values which of course serves almost for all practical purposes.

Group work 1.5

Find the cube of the number 7.89.

a) use rough calculation method.

b) use the numerical table.

c) by calculating the exact value of the number.

d) compare your answer from “a” to “c”.

e) write your generalization.

Example 25: Find the cube of the number 6.95.

Solution:

Do rough calculation and compare your answer with the value obtained from the table.

i. Rough Calculation

$6.95 \approx 7$ and $7^3 = 343$

$(6.95)^3 \approx 343$

ii. Value Obtained from the Table

Step i. Find the row which starts with 6.9

Step ii. Find the column head by 5

Step iii. Read the number, that is the intersection of the row in (i) and the column (ii), therefore $(6.95)^3 = 335.75$

iii. Exact Value

Multiply $6.95 \times 6.95 \times 6.95 = 335.702375$

so $(6.95)^3 = 335.702375$

This examples shows that the result obtained from the numerical tables is an approximation and more closer to the exact value.

Exercise 1I

1. Use the table of cubes to find the cube of each of the following.

a) 3.55	c) 6.58	e) 7.02	g) 9.86	i) 9.90	k) 9.97
b) 4.86	d) 6.95	f) 8.86	h) 9.88	j) 9.94	l) 9.99

1.3.2 Cube Root of a Number

Group work 1.6

Discuss with your group.

1. **In Figure 1.12 to the right, the volume of a cube is 64 m^3. What is the length of each edge?**
2. **Can you define a "cube root" of a number precisely by your own word?**
3. **Find the cube root of 12,167 $\times$ 42,875.**

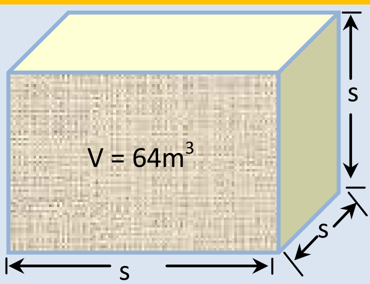


Figure 1.12 Cube

Definition 1.6: **The cube root of a given number is one of the three identical factors whose product is the given number.**

Example 26:

a) $0\times 0\times 0 = 0$, so 0 is the cube root of 0.

b) $5\times5\times5 = 125$, so 5 is the cube root of 125.

c) $\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}=\frac{1}{125}$, so $\frac{1}{5}$ is the cube root of $\frac{1}{125}$.

Note: i) $4^3 = 64$, 64 is the cube of 4 and 4 is the cube root of 64

i.e $\sqrt[3]{64} = 4$.

ii) $\sqrt{}$ is a radical sign.

Index

Or symbolically: $\sqrt[n]{x}$ Radicand

Radical sign

iii) The relation of cubing and extracting cube root can be expressed as follows:

$$a \longrightarrow \boxed{\text{Cubing}} \longrightarrow a^3 = b$$

$$a \longleftarrow \boxed{\text{Cube root}} \longleftarrow a^3 = b$$

iv) a is the cube root of b and written as $a = \sqrt[3]{b}$.

When no index is written, the radical sign indicates a square root.

For example $\sqrt[3]{512}$ is read as "the cube root of 512".

The number 3 is called **the index** and 512 is called **the radicand**.

Cube Roots of Perfect Cubes

Group work 1.7

Discuss with your group

1. Find the cube root of the perfect cubes.

a) $\sqrt[3]{27}$ b) $\sqrt[3]{\frac{1}{27}}$ c) $\sqrt[3]{125}$ d) $\sqrt[3]{-64}$

2. Which of the following are perfect cubes?

$\{42, 60, 64, 90, 111, 125, 133, 150, 216\}$

3. Which of the following are perfect cubes?

$\{3, 6, 8, 9, 12, 27, y^3, y^8, y^9, y^{12}, y^{27}\}$

Note: The following properties of cubes are important: $(ab)^3 = a^3 \times b^3$

and $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$ (where $b \neq 0$).

Thus $(2 \times 2)^3 = 2^3 \times 2^3 = 8 \times 8 = 64$ and $\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$.

A number is called a perfect cube, if it is the cube of a rational number.

Definition 1.7: A rational number x is called a perfect cube if and only if $x = n^3$ for some $n \in \mathbb{Q}$.

Example 27:

$1 = 1^3$, $8 = 2^3$, $27 = 3^3$, $64 = 4^3$ and $125 = 5^3$.

Thus 1, 8, 27 , 64 and 125 are **perfect cubes.**

Note: A perfect cube is a number that is a product of three identical factors of a rational number and its cube root is also a rational number.

Example 28: Find the cube root of each of the following.

a) 216 b) $\frac{1}{8}$ c) -64 d) -27

Solution:

a) $\sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6} = 6$

b) $\sqrt[3]{\frac{1}{8}} = \sqrt[3]{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}} = \frac{1}{2}$

c) $\sqrt[3]{-64} = \sqrt[3]{-4 \times -4 \times -4} = -4$

d) $\sqrt[3]{-27} = \sqrt[3]{-3 \times -3 \times -3} = -3$ -3

Exercise 1J

1. Determine whether each of the following statements is true or false.

   a) $\sqrt[3]{17281} = 26$ b) $\sqrt[3]{\frac{1}{729}} = \frac{1}{90}$ c) $\sqrt[3]{-64} = \pm 4$ d) $\sqrt[3]{\frac{-1}{625}} = \frac{1}{20}$

2. Find the cube root of each of the following.

   a) 0 c) 1000 e) $\frac{1}{729}$ g) $\sqrt[3]{x^3}$ i) $\frac{-1}{1331}$

   b) 343 d) 0. 001 f) $\frac{-1}{9261}$ h) $\frac{0}{27}$

3. Evaluate each of the following.

   a) $\sqrt[3]{-27}$ d) $-\sqrt[3]{\frac{1}{64}}$ f) $\sqrt[3]{-1000}$ h) $-\sqrt[3]{-64}$

   b) $\sqrt[3]{\frac{1}{8}}$ e) $\sqrt[3]{\frac{-8}{27}}$ g) $\sqrt[3]{\frac{-27}{64}}$ i) $\sqrt[3]{\frac{-1}{125}}$

   c) $\sqrt[3]{27}$

Challenge Problem

4. Simplify: a) $5\sqrt{18} - 3\sqrt{72} + 4\sqrt{50}$ b) $\frac{2\sqrt{5} \times 7\sqrt{2}}{\sqrt{14} \times \sqrt{45}}$

5. Simplify the expressions. Assume all variables represent positive rational number.

   a) $\sqrt[3]{\frac{y^5}{27y^3}}$ c) $\sqrt[3]{16a^3}$ e) $\sqrt[3]{\frac{x^5}{x^2}}$ g) $\sqrt[3]{\frac{y^{11}}{y^2}}$

   b) $\sqrt[3]{16z^3}$ d) $\sqrt[3]{\frac{b^4}{27b}}$ f) $\sqrt[3]{15m^4n^{22}}$ h) $\sqrt[3]{20s^{15}t^{11}}$

Table of Cube Roots

The same table which you can used to determine cubes of numbers can be used to find the approximate cube roots, of numbers.

Example 29: Find $\sqrt[3]{64.48}$ from the numerical table.

Solution: Find the value using rough calculations.

$$64.48 \approx 64; \sqrt[3]{64.48} \approx \sqrt[3]{64}$$

$$\approx \sqrt[3]{4 \times 4 \times 4} = 4$$

***Step i*:** Find the number 64. 48 in the body of the table for the relation $y = x^3$.

***Step ii*:** Move to the left on the row containing this number to get 4.0 under x. These are the first two digits of the required cube root of 64. 48.

***Step iii*:** To get the third digits start from 64.48 and move vertically upward and read 1 at the top.

There fore $\sqrt[3]{64.48} \approx 4.01$.

X	0	1	2	3	4	5	6	7	8	9
1.0										
2.0										
3.0										
4.0		64.48								
5.0										
6.0										
7.0										
8.0										
9.0										

Figure 1.13 Tables of cube roots

Example 30:

In Figure 1.14 below, find the exact volume of the boxes.

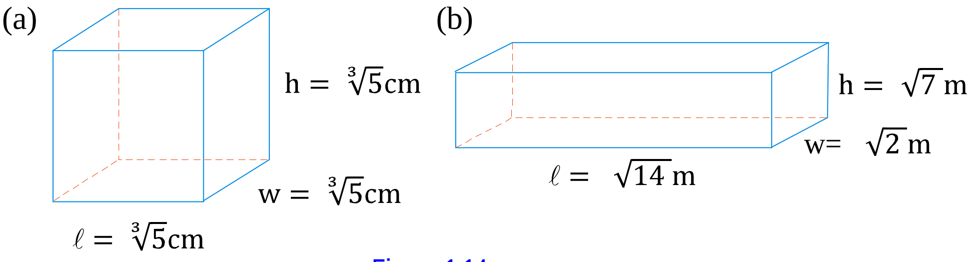


Figure 1.14

Solution

a) $V = \ell \times w \times h$

But the box is a cube, all the side of a cube are equal.

i.e $\ell = w = h = s$

$$V = s \times s \times s = s^3$$

$$V = \sqrt[3]{5}\text{cm} \times \sqrt[3]{5}\text{cm} \times \sqrt[3]{5}\text{cm}$$

$$V = \left(\sqrt[3]{5\text{cm}}\right)^3$$

$$V = \left(5^{\frac{1}{3}}\right)^3$$

$$V = 5\ \text{cm}^3$$

Therefore, the volume of the box is 5cm^3.

b) $V = \ell \times w \times h$

$$V = \sqrt{14}\text{m} \times \sqrt{2}\text{m} \times \sqrt{7}\text{m}$$

$$V = \sqrt{14}\text{m} \times \sqrt{14}\text{m}^2$$

$$V = \left(\sqrt{14 \times 14}\right)\text{m}^3$$

$$V = 14\text{m}^3$$

Therefore, the volume of the box is 14m^3.

Exercise 1k

1. Use the table of cube to find the cube root of each of the following.

a) 32.77	c) 302.5	e) 3114
b) 42.6	d) 329.5	f) 3238

Summary for unit 1

1. *The process of multiplying a number by itself is called* ***squaring*** *the number.*
2. *For each rational number x there is a rational number y ($y \geq 0$) such that $x^2 = y$.*
3. *A square root of a number is one of its two equal factors.*
4. *A rational number x is called a* ***perfect square,*** *if and only if $x = n^2$ for some $n \in \mathbb{Q}$.*
5. *The process of multiplying a number by itself three times is called* ***cubing*** *the number.*
6. *The cube root of a given number is one of the three identical factors whose product is the given number.*
7. *A rational number x is called a* ***perfect cube,*** *if and only if $x = n^3$ for some $n \in \mathbb{Q}$.*
8. index

   $$\sqrt[n]{x}$$

   radicand

   radical sign
9. *The relationship of squaring and square root can be expressed as follows:*

   $$a \xrightarrow{\quad \text{Squaring} \quad} a^2 = b \text{ where } b \geq 0$$

   $$a \xleftarrow{\quad \text{Square root} \quad} a^2$$

   - *a is the square root of b and written as $a = \sqrt{b}$*
10. *The relationship of cubing and cube root can be expressed as follows:*

    $$a \xrightarrow{\quad \textbf{Cubing} \quad} a^3 = b$$

    $$a \xleftarrow{\quad \textbf{Cube root} \quad} a^3 = b$$

    - *a is the cube root of b and written as $a = \sqrt[3]{b}$*

Miscellaneous Exercise 1

1. Determine whether each of the following statements is true or false.

a) $\dfrac{3\sqrt{8}}{2\sqrt{32}} = \dfrac{-3}{4}$

b) $\sqrt{7\frac{1}{9}} = \sqrt{\dfrac{64}{9}}$

c) $\sqrt{\dfrac{2}{5}}\sqrt{\dfrac{125}{8}} = 2.5$

d) $\sqrt{\dfrac{1}{7}} \times \sqrt{63} = \pm 3$

e) $\dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

f) $\sqrt{0.25} = \dfrac{-1}{2}$

g) $\sqrt{0.0036} = 0.06$

2. Simplify each expression.

a) $\sqrt{\dfrac{36}{324}}$

b) $\dfrac{\sqrt{50}}{\sqrt{2}}$

c) $8\sqrt{\dfrac{25}{4}}$

d) $\sqrt{\dfrac{16}{4}}$

e) $2\sqrt{2}\left[\dfrac{3}{\sqrt{2}} + \sqrt{2}\right]$

3. Simplify each expression.

a) $\sqrt{600}$

b) $\sqrt{50} + \sqrt{18}$

c) $\left(5\sqrt{6}\right)^2$

d) $\sqrt{3}\left(\sqrt{3} + \sqrt{6}\right)$

e) $\sqrt{19^2}$

f) $\sqrt{64 + 36}$

g) $\sqrt{2}\left(\sqrt{2} + \sqrt{6}\right)$

h) $\sqrt{2}\left(\sqrt{3} + \sqrt{8}\right)$

4. Simplifying radical expressions (where $x \neq 0$).

a) $\dfrac{\sqrt[3]{32}}{\sqrt[3]{-4}}$

b) $\dfrac{\sqrt[3]{162x^5}}{\sqrt[3]{3x^2}}$

c) $\dfrac{\sqrt{12x^4}}{\sqrt{3x}}$

d) $\sqrt[3]{80n^5}$

e) $\sqrt[3]{p^{17}q^{18}}$

5. Study the pattern and find a and b

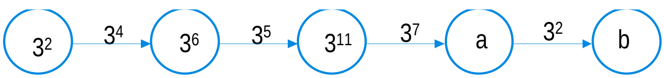


Figure 1.15

6. Study the pattern and find a, b, c and d.

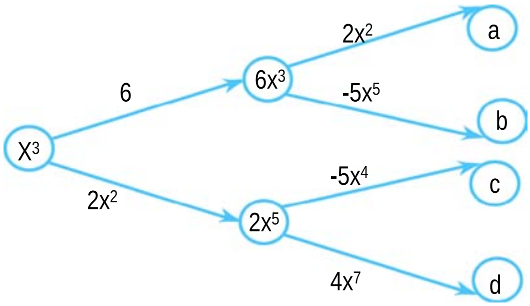


Figure 1.16

7. An amoeba is a single cell animal. When the cell splits by a process called "fission" there are then two animals. In a few hours a single amoeba can become a large colony of amoebas as shown to the right.

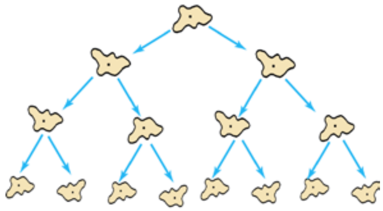


Figure 1.17

Number of splits	Number of amoeba cells
0	1
1	2
2	$4 = 2 \times 2 = 2^2$
3	$8 = 2 \times 2 \times 2 = 2^3$

How many amoebas would there be

a) After 4 splits?
b) After 5 splits?
c) after 6 splits?
d) after 10 splits?

8. Using only the numbers in the circular table, write down, all that are:

a) square numbers
b) cube numbers

Figure 1.18

9. Find the exact perimeter of a square whose side length is $5\sqrt[3]{16}$ cm.

10. The length of the sides of a cubes is related to the volume of the cube according to the formula: $x = \sqrt[3]{V}$.

a) What is the volume of the cube if the side length is 25cm.

b) What is the volume of the cube if the side length is 40 cm.

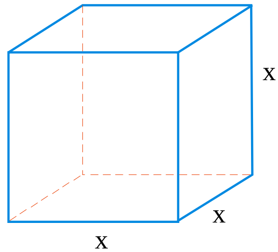


Figure 1.19 Cube

11. In Figure 1.20 to the right find:

a) the surface area of a cube.

b) the volume of a cube.

c) compare the surface area and the volume of a given a cube

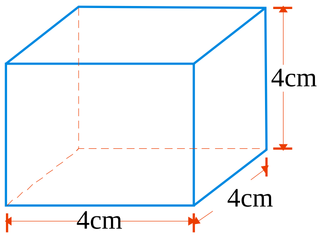


Figure 1.20 Cube

12. Prove that the difference of the square of an even number is multiple of 4.

13. Show that 64 can be written as either 2^6 or 4^3.

14. Look at this number pattern.

$7^2 = 49$

$67^2 = 4489$

$667^2 = 444889$

$6667^2 = 44448889$

This pattern continues.

a) Write down the next line of the pattern.

b) Use the pattern to work out 6666667^2.

15. Find three consecutive square numbers whose sum is 149.

16. Find the square root of $25x^2 – 40xy + 16y^2$.

17. Find the square root of $\frac{64a^2}{9b^2} + 4 + \frac{32a}{3b}$.

18. Find the cube root of $27a^3 + 54a^2b + 36ab^2 + 8b^3$.

UNIT 2

FURTHER ON WORKING WITH VARIABLES

Unit outcomes

After Completing this unit, you should be able to:

- solve life related problems using variables.
- multiply binomial by monomial and determine the product of binomials.
- determine highest common factor of algebraic expressions.

Introduction

By now you are well aware of the importance of variables in mathematics. In this unit you will learn more about variables, specially you will learn about mathematical expression, its component parts and uses of variables in formulas and solving problems. In addition to these you will study special expressions known as binomials and how to perform addition and multiplication on them.

2.1. Further on Algebraic Terms and Expressions

2.1.1 Use of variables in formula

Group Work 2.1

Discuss with your friends

1. What a variable is?
2. Find what number I am left with if
   a. I start with x, double it and then subtract 6.
   b. I start with x, add 4 and then square the result.
   c. I start with x, take away 5, double the result and then divide by 3.
   d. I start with w, subtract x and then square the result.
   e. I start with n add p, cube the result and then divide by a.
3. Translate the following word problems in to mathematical expression.
   a. Eighteen subtracted from 3.
   b. The difference of -5 and 11.
   c. Negative thirteen subtracted from – 10.
   d. Twenty less than 32.
4. Describe each of the following sets using variables.
   a. The set of odd natural numbers.
   b. The solution set of $3x - 1 \geq 4$.
   c. The solution set of $x + 6 = 24$.
   d. The set of all natural number less than 10.

Definition 2.1: ***A variable*** **is a symbol or letter such as x, y and z used to represent an unknown number (value).**

Example 1: Describe each of the following sets using variables.

a. The solution set of $3x - 5 > 6$.

b. The solution set of $2x + 1 = 10$.

Solution

a) $3x - 5 > 6$ .... Given inequality

$3x - 5 + 5 > 6 + 5$ .... Adding 5 from both sides

$3x > 11$ .... Simplifying

$\frac{3x}{3} > \frac{11}{3}$ .... Dividing both sides by 3

$x > \frac{11}{3}$

The solution set of $3x - 5 > 6$ is $\left\{x : x > \frac{11}{3}\right\}$.

b) $2x + 1 = 10$ .... Given equation.

$2x + 1 - 1 = 10 - 1$ ......Subtracting 1 from both sides.

$2x = 9$ .... Simplifying.

$\frac{2x}{2} = \frac{9}{2}$ .... Dividing both sides by 2.

$x = \frac{9}{2}$

The solution set of $2x + 1 = 10$ is $x = \frac{9}{2}$ or $S.S = \left\{\frac{9}{2}\right\}$

Example 2: Find the perimeter of a rectangle in terms of its length ℓ and width w.

Solution Let P represent the perimeter of the rectangle.

Then $P = AB + BC + CD + DA$

$= \ell + w + \ell + w$

$= 2\ell + 2w$

$= 2(\ell + w)$

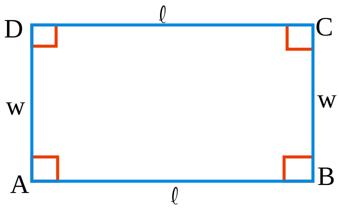


Figure 2.1

Example 3: The volume of a rectangular prism equals the product of the numbers which measures of the length, the width and the height. Formulate the statement using variables.

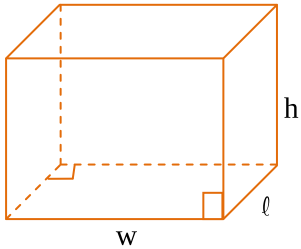


Figure 2.2 A rectangular prism

Solution let ℓ represent the length, w the width and h the height of the prism. If V represents the volume of the prism, then

$V = \ell \times w \times h$

$V = \ell wh$

Example 4: Express the area of a triangle in terms of its base 'b' and altitude 'h'.

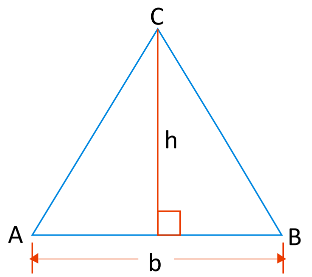


Figure 2.3 triangle

Solution Let "b" represent the base and "h" the height of the triangle.

$A = \frac{1}{2}bh$

Example 5: The area of a trapezium (see Figure 2.4) below can be given by the formula $A = \frac{1}{2}(b_1 + b_2)$ where A = area, h = height, b_1 = upper base and b_2 = lower base. If the area is 170 cm^2, height 17cm and b_2 = 12cm then:

a) Express b_1 in terms of the other variables in the formula for A.

b) Use the equation you obtained to find b_1.

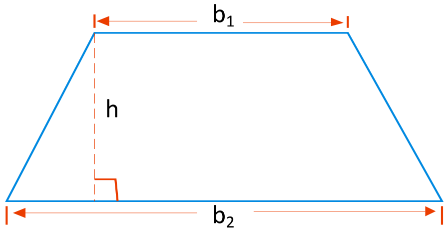


Figure 2.4 Trapezium

Solution

a) $\frac{1}{2}h(b_1 + b_2) = A$ …. Given equation

$h(b_1 + b_2) = 2A$ ….. Multiplying A by 2

$b_1 + b_2 = \frac{2A}{h}$ ….. Dividing both sides by h

$b_1 = \frac{2A}{h} - b_2$ ….. Subtracting b_2 from both sides

$b_1 = \frac{2A - b_2h}{h}$ …… Simplifying

b) For (a) above we have

$b_1 = \frac{2A - b_2h}{h}$

$b_1 = \frac{2(170) - 17(12)}{17}$

$b_1 = \frac{17(20-12)}{17}$

$b_1 = 8cm$

Therefore the upper base (b_1) is 8cm.

✓ **Check:** $A = \frac{1}{2}h(b_1 + b_2)$ when $b_1 = 8cm$

$170cm^2 \stackrel{?}{=} \frac{1}{2}(17cm)\,(8cm + 12cm)$

$170\ cm^2 \stackrel{?}{=} \frac{17}{2}\ cm\ (20cm)$

$170\ cm^2 = 170\ cm^2$ (True)

Exercise 2A

Solve each of the following word problems.

1. The perimeter of a rectangular field is 1000m. If the length is given as x, find the width in terms of x.
2. Find
   i) The perimeter of a square in terms of its side of length "s" unit.
   ii) The area of a square interms of its side of length "s" units.
3. Express the volume of the cube in Figure 2.5.

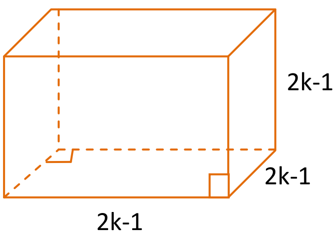


Figure 2.5 Cube

4. The area of a trapezoid is given by the formula $A = \left(\frac{b_1 + b_2}{2}\right)h$ then give the height h interms of its bases b_1 & b_2.
5. A man is 8x years old now. How old he will be in:
   a. 10 years time? b. 6x years time? c. 5y years time?
6. How many days (d) are there in the given number of weeks (w) below?
   a. 6 weeks
   b. 104 weeks
   c. y weeks
   d. 14 weeks

2.1.2 Variables, Terms and Expressions

Activity 2.1

Discuss with your teacher before starting the lesson.

1. What do we mean by like terms? Given an example.
2. Are $7a^3$, $5a^2$ and $12a$ like terms? Explain.
3. What is an algebraic expression?
4. What is a monomial?
5. What is a binomial?
6. What is a trinomial?
7. What are unlike terms? Give an example.

Definition 2.2: **Algebraic expressions are formed by using numbers, letters and the operations of addition, subtraction, multiplication, division, raising to power and taking roots.**

Some examples of algebraic expressions are:

$x + 10$, $y – 16$, $2x^2 + 5x – 8$, $x – 92$, $2x + 10$, etc.

Note:

i. An algebraic expression that contains variables is called an expression in certain variables. For examples the expression $7xy + 6z$ is an algebraic expression with variables x, y and z.

ii. An algebraic expression that contains no variable at all is called constant. For example, the algebraic expression $72 - 16\pi$ is constant.

iii. The terms of an algebraic expression are parts of the expression that are connected by plus or minus signs.

Examples 6: List the terms of the expression $5x^2 – 13x + 20$.

Solution: The terms of the expression $5x^2 – 13x + 20$ are $5x^2$, $- 13x$, and 20.

Definition 2.3: **An algebraic expression in algebra which contains one term is called a monomial.**

Example 7: $8x$, $13a^2b^2$, $\frac{-2}{3}$, $18xy$, $0.2a^3b^3$ are all monomial.

Definition 2.4: **An algebraic expression in algebra which contains two terms is called a binomial.**

Examples 8: $2x + 2y$, $2a – 3b$, $5p^2 + 8$, $3x^2 + 6$, $n^3 – 3$ are all binomial.

Definition 2.5: **An algebraic expression in algebra which contains three terms is called a trinomial.**

Examples 9: $4x^2 + 3x + 10$, $3x^2 – 5x + 2$, $ax^2 + bx + c$ are all trinomial.

Definition 2.6: Terms which have the same variables, with the corresponding variables are raised to the same powers are called like terms; other wise called unlike terms.

For example:

Like terms	Unlike terms
34xy and -8xy	12xy and 6x …… Different variables.
$18p^2q^3$ and p^2q^3	$8p^2q^3$ and $16p^3q^2$ …Different power
5w and 6w	10w and 20 …… Different variables.
7 and 20	14 and 10a…… Different variables.

Example 10: Which of the pairs are like terms: 80ab and 70b or $4c^2d^2$, and $-6c^2 d^2$.

Solution $4c^2d^2$ and $-6c^2d^2$ are like terms but 80 ab and 70 b are unlike terms.

Note:

i. Constant terms with out variables, (or all constant terms) are like terms.
ii. Only like terms can be added or subtracted to form a more simplified expression.
iii. Adding or subtracting like terms is called combining like terms.
iv. If an algebraic expression contains two or more like terms, these terms can be combined into a single term by using distributive property.

Example 11: Simplify by collecting like terms.

a. $18x + 27 – 6x – 2$
b. $18k – 10k – 12k + 16 + 7$

Solution

a. $18x + 27 – 6x – 2$
   $= 18x - 6x + 27 – 2$ ……. Collecting like terms
   $= 12x + 25$ ……………… Simplifying
b. $18k – 10k – 12k + 16 + 7$
   $= 18k – 22k + 16 + 7$ ….. Collecting like terms
   $= -4k + 23$ …….. ……… Simplifying

Example 12: Simplify the following expressions

a. $(6a + 9x) + (24a – 27x)$
b. $(10x + 15a) – (5x + 10y)$
c. $– (4x – 6y) – (3y + 5x) – 2x)$

Solution

a. $(6a + 9x) + (24a – 27x)$
   $= 6a + 9x + 24a – 27x$ ……. Removing brackets
   $= 6a + 24a + 9x – 27x$ ……. Collecting like terms
   $= 30a – 18x$ ….. Simplifying
b. $(10x + 15a) – (5x + 10y)$

= 10x + 15a – 5x – 10y ……. Removing brackets
= 10x – 5x + 15a – 10y ……. Collecting like terms
= 5x + 15a – 10y …… . . . . . .Simplifying

c. –(4x – 6y) – (3y + 5x) – 2x
= -4x + 6y – 3y – 5x – 2x ……Removing brackets
= -4x – 5x – 2x + 6y – 3y ……Collecting like terms
= -11x + 3y ………………… Simplifying

Group work 2.2

1. Three rods A, B and C have lengths of (x + 1) cm; (x + 2) cm and (x – 3)cm respectively, as shown below.

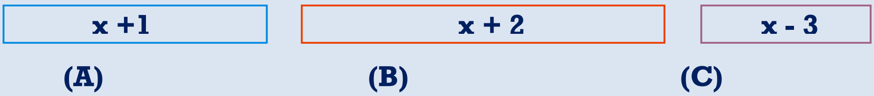


Figure 2.6

In the figures below express the length ℓ interms of x.

Give your answers in their simplest form.

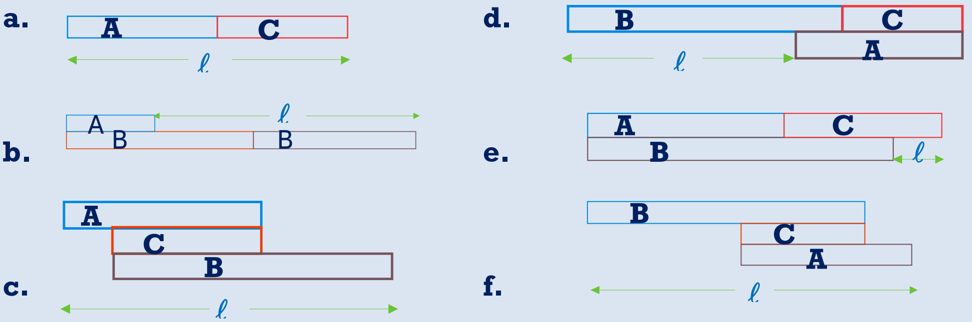


Figure 2.7

2. When an algebraic expression was simplified it became 2a + b.

a. Write down as many different expressions as you can which simplify to 2a + b.

b. What is the most complex expression you can think of that simplifies to 2a + b?

c. What is the simplest expression you can think of that simplifies to 2a + b?

Note: An algebraic formula uses letters to represent a relationship between quantities.

Exercise 2B

1. Explain why the terms $4x$ and $4x^2$ are not like terms.
2. Explain why the terms $14w^3$ and $14z^3$are not like terms.
3. Categorize the following expressions as a monomial, a binomial or a trinomial.

a. 26	d. $16x^2$	g. $20w^4 – 10w^2$
b. $50\,bc^2$	e. $10a^2 + 5a$	h. $2t – 10t^4 – 10a$
c. $90 + x$	f. $27x + \frac{3}{2}$	i. $70z + 13z^2 – 16$

4. Work out the value of these algebraic expressions using the values given.
   a. $2(a + 3)$ if $a = 5$
   b. $4(x + y)$ if $x = 5$ and $y = -3$
   c. $\frac{7-x}{y}$ if $x = -3$ and $y = -2$
   d. $\frac{2a + b}{c}$ if $a = 3$, $b = 4$ and $c = 2$
   e. $2(b + c)^2 – 3(b – c)^2$ if $b = 8$ and $c = -4$
   f. $(a + b)^2 + (a + c)^2$ if $a = 2$, $b = 8$ and $c = -4$
   g. $c(a + b)^3$ if $a = 3$, $b = 5$ and $c = 40$

Challenge Problems

5. Solve for d if $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ if $x_1 = 3$, $y_1 = 4$ and $x_2 = 12$, $y_2 = 37$.
6. $y\,\frac{[3x+6y(x-20)]}{2x+12}$ if $x = 5$ and $y = \frac{1}{2}$
7. Collect like terms together.

a. $xy + ab – cd + 2xy – ab + dc$	d. $5 + 2y + 3y^2 – 8y – 6 + 2y^2 + 3$
b. $3x^2 + 4x + 6 – x^2 – 3x – 3$	e. $6x^2 – 7x + 8 – 3x^2 + 5x – 10$
c. $3y^2 – 6x + y^2 + x^2 + 7x + 4x^2$	f. $2x^2 – 3x + 8 + x^2 + 4x + 4$

2.1.3 Use of Variables to Solve Problems

Activity 2.2

Discuss with your teacher

1. Prove that the sum of two even numbers is an even numbers.
2. If the perimeter of a rectangle is 120cm and the length is 8cm more than the width, find the area.
3. The sum of three consecutive integers is 159. What are the integers?
4. The height of a ballon from the ground increases at a steady rate of x metres in t hours. How far will the ballon rise in n hours?

In this topic you are going to use variables to solve problems involving some unknown values and to prove a given statement.

? What is a proof?

A **proof** is an argument to show that a given statements is true. The argument depends on known facts, such as definitions, postulates and proved theorems.

Example13: (**Application involving consecutive integers**)

The sum of two consecutive odd integers is -188. Find the integers.

Solution: Let x represent the first odd integer, hence

x + 2 represents the second odd integer.

(First integer) + (Second integer) = total ……. Write an equation in words.

x + (x + 2) = -188 …… Write a mathematical equation

$x + x + 2 = -188$

$2x + 2 = -188$

$2x = -190$

$\frac{2x}{2} = \frac{-190}{2}$

$x = -95$

Therefore the integer are -95 and -93.

Example14: (**Application involving Ages**)

The ratio of present ages of a mother and her son is 12 : 5. The mother's age, at the time of birth of the son was 21 years. Find their present ages.

$\left(\text{Hint: } \frac{x}{y} = \frac{12}{5}\right)$

Solution:

Let x be the present age of the mother and y be that of her son.

Thus x:y = 12:5 or $\frac{x}{y} = \frac{12}{5}$

$5x - 12y = 0$ …... Equation 1

$x - y = 21$ …... Equation 2

From equation 2, we get $x = 21 + y$ …… Equation 3

Substituting equation 3 in to equation 1, we will get:

$5(21 + y) - 12y = 0$

$105 + 5y - 12y = 0$

$105 - 7y = 0$

$105 = 7y$

$y = 15$

Thus $x = 21 + y$

$x = 21 + 15 \Rightarrow x = 36$

Therefore, the present ages of a mother and her son are 36 years and 15 years respectively.

Exercise 2C

Solve each of the following word problems.

1. A 10 meter piece of wire is cut in to two pieces. One piece is 2 meters longer than the other. How long are the pieces?
2. The perimeter of a college basket ball court is 96 m and the length is 14m more than the width. What are the dimensions?
3. Ten times the smallest of three consecutive integers is twenty two more than three times the sum of the integers. Find the integers.
4. The surface area "S" of a sphere of radius r is given by the formula: $S = 4\pi r^2$.
   Find (i) the surface area of a sphere whose radius is 5 cm.
   (ii) the radius of a sphere whose surface area is $17\frac{1}{9}$ cm^2.
5. By what number must be 566 be divided so as to give a quotient 15 and remainder 11?
6. I thought of a number, doubled it, then added 3. The result multiplied by 4 came to 52. What was the number I thought of ?

7. One number is three times another, and four times the smaller added to five times the greater amounts to 133; find them.

Challenge Problems

8. If a certain number is increased by 5, one – half of the result is three – fifths of the excess of 61 over the number. Find the number.
9. Divide 54 in to two parts so that four times the greater equals five times the less.
10. Prove that the sum of any 5 consecutive natural numbers is divisible by 5.

2.2 Multiplication of Binomials

In grade seven you have studied about certain properties of multiplication and addition such as the commutative and associative properties of addition and multiplication and the distributive of multiplication over addition. In this sub–unit you will learn how to perform multiplication of monomial by binomial and multiplication of binomial by binomial.

2.2.1 Multiplication of Monomial by Binomial

Activity 2.3

Discuss with your friends /partners/

1. Multiply 4a by 2ab
2. Multiply 4b by (2ab + 6b)
3. Multiply 6b by (3a + 15b)
4. Multiply 7ab by (3ab – 6a)

You begin this topic, let us look at some examples:

Example 15: Multiply 2x by 4yz

Solution:

$$2x \times 4yz = 2 \times x \times 4 \times y \times z$$
$$= (2 \times 4)(x \times y \times z)$$
$$= 8\,xyz$$

Example 16: Multiply $4c^2$ by $(16\,abc – 5a^2\,bc)$

Solution:

$$4c^2 \times (16abc - 5a^2bc)$$
$$= (4c^2 \times 16abc) - (4c^2 \times 5a^2bc)$$
$$= (4 \times 16 \times c^2 \times c \times a \times b) - (4 \times 5 \times c^2 \times c \times a^2 \times b)$$
$$= 64\,c^3\,ab - 20\,c^3\,a^2b$$

Example 17: Multiply 4rt (5pq – 3pq)

Solution:

$4rt \times (5pq - 3pq)$
$= (4rt \times 5pq) - (4rt \times 3pq)$
$= (4 \times 5 \times r \times t \times p \times q) - (4 \times 3 \times r \times t \times p \times q)$
$= (20rt\,pq - 12\,rt\,pq)$
$= (20\text{-}12)\,rt\,pq$
$= 8pqrt$

? Do you recall the properties used in examples 15, 16, and 17 above?

1. Distributive properties

Group Work 2.3

1. In Figure 2.8 below, find the area of the shaded region.

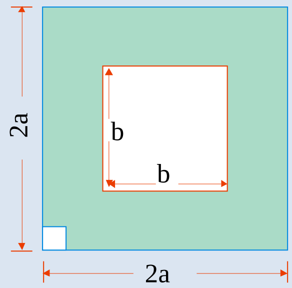


Figure 2.8

2. If the area of a rectangle is found by multiplying the length times the width, express the area of the rectangle in Figure 2.9 in two ways to illustrate the distributive property for a(b + c).

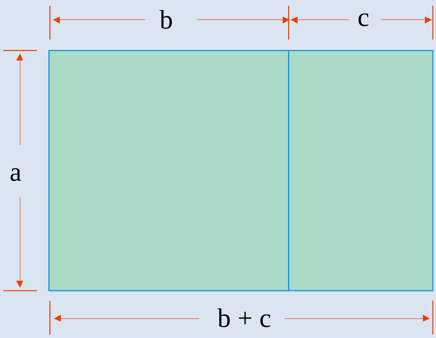


Figure 2.9

3. Express the shaded area of the rectangle in Figure 2.10 in two ways to illustrate the distributive property for a (b – c).

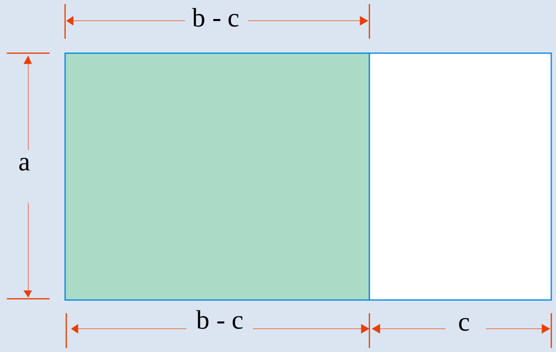


Figure 2.10

4. In Figure 2.11, find the area of each rectangle.

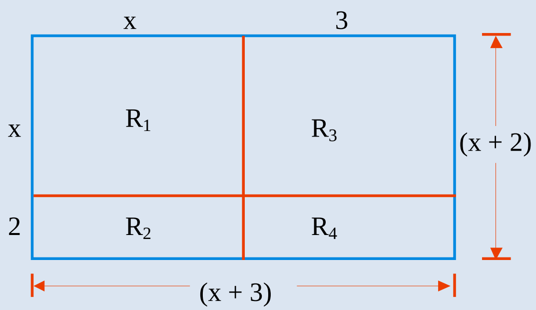


Figure 2.11

Consider the rectangle in Figure 2.12 which has been divided in to two smaller ones:

The area A of the bigger rectangle is given by: $A = a(b + c)$.

The area of the smaller rectangles are given by $A_1 = ab$ and $A_2 = ac$; but the sum of the areas of the two smaller rectangles are given by,

$A_1 + A_2$ gives the area A of the bigger rectangle:

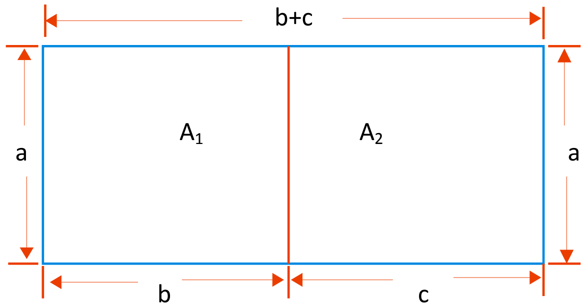


Figure 2.12 Rectangle

That means: $A = A_1 + A_2$

$a(b + c) = ab + ac$

This suggests that $a(b + c) = a \times b + a \times c$ the factor out side the bracket multiply each number in the bracket, this process of removing the bracket in a product is known as **expansion**.

Similarly consider another rectangle as in Figure 2.13 below.

Area of the shaded region = area of the bigger rectangle–area of the un shaded region.

Therefore, $a(b – c) = ab – ac$.

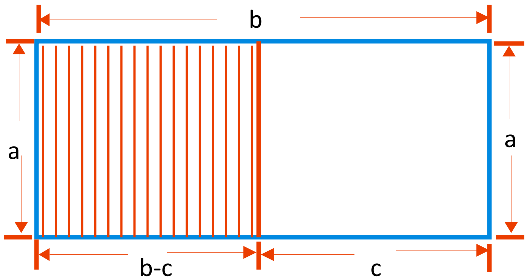


Figure 2.13 Rectangle

You have seen that the above two examples on area of rectangle, this could be generalized as in the following way:

Note: For any rational numbers a, b, and c

a. $a(b + c) = ab + ac$ b. $a(b – c) = ab – ac$

These two properties are called the distributive properties of multiplication over Addition (subtraction).

Exercise 2D

1. Expand these expressions by using the distributive properties to remove the brackets in and then simplify.

1. $2(a + b) + 3(a + b)$
2. $5(2a – b) + 49(a + b)$
3. $4(5a+c) + 2(3a – c)$
4. $5(4t – 3s) + 8(3t + 2s)$
5. $5(3z + b) + 4(b – 2z)$
6. $7(2d + 3e) + 6(2e – 2d)$
7. $3(p + 2q) + 3(5p – 2q)$
8. $5(5q + 4h) + 4(h – 5q)$
9. $6(p + 2q + 3r) + 2(3p – 4q + 9r)$
10. $2(a + 2b – 3c) + 3(5a – b + 4c) + 4(a + b + c)$

Challenge Problems

11. Remove the brackets and simplify.

a) $(x + 1)^2 + (x + 2)^2$
b) $(y - 3)^2 + (y - 4)^2$
c) $(x - 2)^2 + (x + 4)^2$
d) $(x + 2)^2 - (x - 4)^2$
e) $(2x + 1)^2 + (3x + 2)^2$
f) $(2x - 3)^2 + (5x + 4)^2$

2.2.2 Multiplication of Binomial by Binomial

Activity 2.4

Discuss with your friends / partners

Find the following products

1. $(2x + 8)(3x - 6)$
2. $(5a + 4)(4a + 6)$
3. $(2x - 8)(2x + 8)$
4. $(2x - 10)(x - 8)$
5. $(2a^2 - ab)(20 + x)$
6. $(3x^2 + 2x - 5)(x - 1)$

Sometimes you will need to multiply brackets expressions. For example $(a + b)(c + d)$.

This means $(a + b)$ multiplied by $(c + d)$ or $(a + b) \times (c + d)$.

Look at the rectangles 2.14 below.

The area 'A' of the whole rectangle is $(a + b)(c + d)$. It is the same as the sum of the areas of the four rectangle so: $A = A_1 + A_2 + A_3 + A_4$

$$(a + b)(c + d) = ac + ad + bc + bd$$

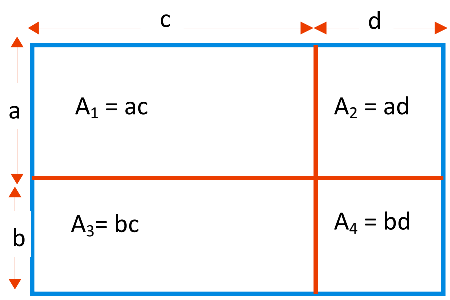


Figure 2.14 Rectangle

Notice that each term in the first brackets is multiplied by each term in the second brackets:

$a \times d$

$a \times c$

$(a + b)\ (c + d)$

$b \times c$

$b \times d$

You can also think of the area of the rectangle as the sum the areas of two separate part (the upper two rectangles plus the lower two rectangle) see Figure 2.15:

Thus, $(a + b)\ (c + d) = a\ (c + d) + b(c + d)$

Think of multiplying each term in the first bracket by the whole of the second bracket. These are two ways of thinking about the same process. The end result is the same. This is called **multiplying out** the brackets.

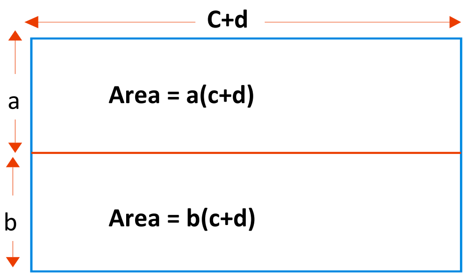


Figure 2.15 Rectangle

This process could be described as follows.

Note: If (a + b) and (c +d) are any two binomials their product

$(a + b) \times (c + d)$ is defined as:

$(a + b) \times (c + d) = ac + ad + bc + bd$

Example18: Multiply (4x + 4) by (3x + 8)

Solution

$$(4x + 4)(3x + 8) = (4x \times 3x) + (4x \times 8) + (4 \times 3x) + (4 \times 8)$$
$$= 12x^2 + 32x + 12x + 32$$
$$= 12x^2 + 44x + 32$$

Example 19: Multiply (2x + 10) by (3x - 6)

Solution

$$(2x + 10)(3x - 6) = (2x \times 3x) - (6 \times 2x) + (10 \times 3x) - (6 \times 10)$$
$$= 6x^2 - 12x + 30x - 60$$
$$= 6x^2 + 18x - 60$$

Example20: Multiply (2x - 3) by (4x - 12)

Solution

$$(2x - 3)(4x - 12) = (2x \times 4x) - (12 \times 2x) - (3 \times 4x) + (3 \times 12)$$
$$= 8x^2 - 24x - 12x + 36$$
$$= 8x^2 - 36x + 36$$

In the multiplication of two binomials such as those shown in example 20 above, the product $2x \times 4x = 8x^2$ and $-3 \times -12 = 36$ are called **end products**. Similarly, the product $-12 \times 2x = -24x$ and $-3 \times 4x = -12x$ are called **cross product**. Thus the product of any two binomials could be defined as the sum of the **end products** and the **cross products**. The sum of the cross products is written in the middle.

Example21: Multiply (4x - 6) by (4x + 10)

Solution

end product

$$(4x - 6)(4x + 10) = (4x \times 4x) + (4x \times 10) - (6 \times 4x) - (6 \times 10)$$

Cross product

$$= 16x^2 + 40x - 24x - 60$$
$$= 16x^2 + 16x - 60$$

Example22: Multiply (4x-10) by (6x-2)

Solution:

$$(4x - 10)(6x - 2) = \overbrace{(4x \times 6x)-\underbrace{(2\times 4x)-(10\times 6x)}_{\text{cross product}}+(10\times 2)}^{\text{end product}}$$

$$= 24x^2-8x-60x+20$$

$$= 24x^2-68x+20$$

Exercise 2E

Find the products of the following binomials.

1. $(2x + 2y)(2x - 2y)$
2. $(3x + 16)(2x - 18)$
3. $(-4x - 6)(-20x + 10)$
4. $-5\,[(4x+y)(3x+2b)]$
5. $\left(\frac{3}{2} - \frac{2}{3}x\right)(2x + 1)$
6. $\left(\frac{x}{8}+\frac{x}{8}\right)\left(\frac{x}{8} - \frac{x}{4}\right)$
7. $\left(\frac{3}{2}x+ \frac{4}{3}x\right)\left(\frac{2}{3}x+ \frac{3}{5}x\right)$
8. $\left(\frac{4}{5}ab- \frac{3}{5}ab\right)\left(-4ab-\frac{3}{2}ab\right)$
9. $\left(\frac{2}{5}ab+ \frac{3}{5}a^2b^2\right)\left(\frac{3}{7}a^2b^2+ \frac{3}{7}a^2b^2\right)$

Challenge Problems

10. $(2x^2 + 4x - 6)(x^2 + 4)$
11. $(2x^2 - 4x - 6)\left(\frac{3}{2}x^2-6\right)$
12. $(4x^2 + 4x - 10)(5x - 5)$

2.3 Highest Common Factors

Activity 2.5

Discuss with your teacher before starting the lesson.

1. Define and explain the following key terms:
   a. Factorizing a number
   b. Prime factorization
2. Find the HCF of the following.
   a. 72 and 220
   b. 36, 48 and 72
   c. 120, 150 and 200

3. Find the HCF of the following:
   a. $20xyz$ and $18x^2z^2$
   b. $5x^3y$ and $10xy^2$
   c. $3a^2b^2$, $6a^3b$ and $9a^3b^3$
4. Factorize the following expressions.
   a. $\frac{3}{19}ac - \frac{1}{19}ad$
   b. $x(2b + 3) + y(2b + 3)$
   c. $\frac{5a^2b^2}{4} + \frac{15}{6}a^4b^2$
   d. $a^2(c+2d)-b^2(c+2d)$

Factorizing

This unit is devoted to the method of describing an expression is called **Factorizing**. To factorize an integer means to write the integer as a product of two or more integers. To factorize a monomial or a Binomial means to express the monomial or Binomial as a product of two or more monomial or Binomials. In the product $2\times5 = 10$, for example, 2 and 5 are factors of 10. In the product $(3x + 4)(2x) = 6x^2 + 8x$, the expressions $(3x + 4)$ and $2x$ are factors of $6x^2 + 8x$.

Example 23: Factorize each monomial in to its linear factors with coefficient of prime numbers.

a. $15x^3$ b. $25x^3$

Solution:

a. $15x^3 = (3\times 5) \times (x \times x \times x)$
$= (3x) \times (5x) \times (x)$.

b. $25x^3 = (5 \times 5) \times (x \times x \times x)$
$= (5x) \times (5x) (x)$.

Example 24: Factorize each of the following expressions.

a. $6x^2 + 12$ b. $5x^4 + 20x^3$

Solution:

a. $6x^2 + 12 = 6x^2 + 6 \times 2$
$= 6(x^2 + 2)$

b. $5x^4 + 20x^3 = 5x^3 \times x + 5x^3 \times 4$
$= 5x^3(x + 4)$

Exercise 2F

Factorize each of the following expressions.

1. $2x^2 + 6x$
2. $18xy^2 – 12xy^3$
3. $5x^3y + 10xy^2$
4. $16a^2b + 24ab^2$
5. $12ab^2c^3 + 16ac^4$
6. $3a^4b – 5bc^3$
7. $6x^4yz + 15x^3y^2z$
8. $8a^2b^3c^4 – 12a^3b^2c^3$
9. $8xy^2 + 28xyz – 4xy$
10. $-10mn^3 + 4m^2n – 6mn^2$

Challenge Problems

11. $7a^2b^3 + 5ab^2 + 3a^2b$
12. $2a^3b^3 + 3a^3b^2 + 4a^2b$
13. $-30abc + 24abc – 18a^2b$
14. $16x^4 – 24x^3 + 32x^2$
15. $10x^3 + 25x^2 + 15x$

Highest common factor of two integers

Group Work 2.4

Discuss with your group

For Exercise 1 – 4 factor out the HCF.

1. $15x^2 + 5x$

2. $5q^4 – 10q^5$

3. $y(5y + 1) – 9(5y + 1)$

4. $5x(x – 4) – 2(x – 4)$

You begin the study of factorization by factoring integers. The number 20 for example can be factored as 1×20, 2×10, 4×5 or $2\times2\times5$. The product $2\times2\times5$ (or equivalently $2^2\times5$) consists only of prime numbers and is called the **prime factorization**.

The **highest common factor** (denoted by HCF) of two or more integers is the highest factor common to each integer. To find the highest common factor of two integers, it is often helpful to express the numbers as a product of prime factors as shown in the next example.

Example 25: Find the highest common factor of each pair of integers.

a. 24 and 36

b. 105 and 40

Solution:

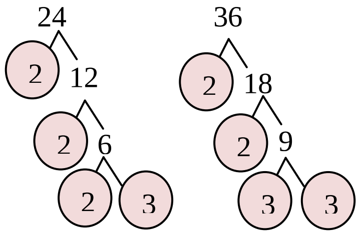
First find the prime factorization of each number by multiplication or by factor tree method.

a. i. by multiplication

Factors of 24 = $2 \times 2 \times 2 \times 3$

Factors of 36 = $2 \times 2 \times 3 \times 3$

ii. By using factor trees



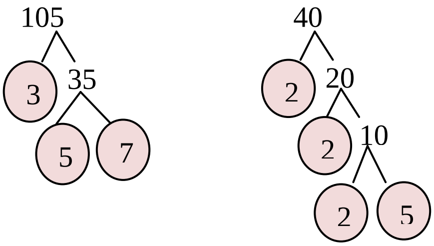
The numbers 24 and 36 share two factors of 2 and one factor of 3. Therefore, the highest common factor is $2\times2\times3 = 12$

b. i. By multiplication

Factors of 105 = $3 \times 7 \times 5$

Factors of 40 = $2\times2\times 2 \times 5$

ii. By using factor trees



Therefore, the highest common factors is 5.

Highest common factor (HCF) of two or more monomials

Example 26: Find the HCF among each group of terms.

a. $7x^3$, $14x^2$, $21x^4$

b. $8c^2d^7e$, $6c^3d^4$

Solution:

List the factors of each term.

a) $7x^3 = 7 \times x \times x \times x$

$14x^2 = 2 \times 7 \times x \times x$

$21x^4 = 3 \times 7 \times x \times x \times x \times x$

Therefore, the HCF is $7x^2$.

b) $\left.\begin{aligned} 8c^2d^7e &= 2^3c^2d^7e \\ 6c^3d^4 &= 2\times 3c^3d^4 \end{aligned}\right\}$ The common factors are the common powers of 2, c and d appearing in both factorization to determine the HCF we will take the common least powers. Thus

$\left.\begin{aligned} &\text{The lowest power of 2 is : } 2^1 \\ &\text{The lowest power of c is : } c^2 \\ &\text{The lowest power of d is : } d^4 \end{aligned}\right\}$

Therefore, the HCF is $2c^2d^4$.

Example 27: Find the highest common factor between the terms:

$3x(a + b)$ and $2y(a + b)$

Solution

$\left.\begin{aligned} &3x\,(a + b) \\ &2y(a + b) \end{aligned}\right\}$ The only common factor is the binomial $(a + b)$.

Therefore, the HCF is $(a + b)$.

Factorizing out the highest common factor

Factorization process is the reverse of multiplication process. Both processes use the distributive property: $ab + ac = a\,(b + c)$

Example 28: Multiply: $5y(y^2 + 3y + 1)$

$= 5y\,(y^2) + 5y(3y) + 5y(1)$

$= 5y^3 + 15y^2 + 5y$

Factor: $5y^3 + 15y^2 + 5y$

$= 5y(y^2) + 5y(3y) + 5y(1)$

$= 5y\ (y^2 + 3y + 1)$

Example 29: Find the highest common factors

a. $6x^2 + 3x$ b. $15y^3 + 12y^4$ c. $9a^4b - 18a^5b + 27a^6b$

Solution a. The HCF of $6x^2 + 3x$ is $3x$ … Observe that $3x$ is a common factor.

$6x^2 + 3x = (3x \times 2x) + (3x \times 1)$ … Write each term as the product of $3x$ and another factor.

$= 3x\ (2x + 1)$ …… Use the distributive property to factor out the HCF.

Therefore, the HCF of $6x^2 + 3x$ is $3x$.

✓ Check: $3x\ (2x + 1) = 6x^2 + 3x$

b. The HCF of $15y^3 + 12y^4$ is $3y^3$ …Observe that $3y^3$ is a common factor.

$15y^3 + 12y^4 = (3y^3 \times 5) + (3y^3 \times 4y)$ …. Write each term as the product of $3y^3$ and another factor.

$= 3y^3(5 + 4y)$ ……Use the distributive property to factor out the HCF.

Therefore, the HCF of $15y^3 + 12y^4$ is $3y^3$.

c. $9a^4b - 18a^5b + 27a^6b$ is $9a^4b$ …Observe that $9a^4b$ is a common factor.

$= (9a^4b \times 1) - (9a^4b \times 2a) + (9a^4b \times 3a^2)$ …… Write each term as the product of $9a^4b$ and another factor.

$= 9a^4b(1 - 2a + 3a^2)$ …… Use the distributive property to factor out the HCF.

Therefore the HCF of $9a^4b - 18a^5b + 27a^6b$ is $9a^4b$.

Factorizing, out a binomial factor

The distributive property may also be used to factor out a common factor that consists of more than one term such as a binomial as shown in the next example.

Example 30: Factor out the highest common factor:

$2x(5x + 3) - 5(5x + 3)$

Solution

$2x(5x + 3) - 5(5x + 3)$ …….. The Highest common factor is the binomial $5x + 3$

$= (5x + 3) \times (2x) - (5x + 3) \times (5)$ ……… Write each term as the product of $(5x + 3)$ and another factor.

$= (5x + 3)(2x - 5)$ ….. Use the distributive property to factor out the HCF.

✓Check: $(5x + 3)(2x - 5) = (5x + 3)(2x) + (5x + 3)(-5)$

$= 2x(5x + 3) - 5(5x + 3)$

Exercise 2G

1. Find the highest common factor among each group of terms.
   a. $-8xy$ and $20y$
   b. $20xyz$ and $15yz^2$
   c. $6x$ and $3x^2$
   d. $2ab$, $6abc$ and $4a^2c$
   e. $3x^2y$, $6xy^2$ and $9xyz$
   f. $15a^3b^2$ and $20ab^3c$
   g. $6ab^4c^2$ and $12a^2b^3cd$
2. Find the highest common factor of the pairs of the terms given below.
   a. $(2a - b)$ and $3(2a - b)$
   b. $7(x - y)$ and $9(x - y)$
   c. $14(3x + 1)^2$ and $7(3x + 1)$
   d. $a^2(x + y)$ and $a^3(x + y)^2$
   e. $21x(x + 3)$ and $7x^2(x + 3)$
   f. $5y^3(y - 2)$ and $-20y(y - 2)$
3. Factor out the highest common factor.
   a. $13(a + 6) - 4b(a + 6)$
   b. $7(x^2 + 2) - y(x^2 + 2)$
   c. $8x(y^2 - 2) + (y^2 - 2)$
   d. $4(x + 5)^2 + 5x(x + 5) - (x + 5)$
   e. $6(z - 1)^3 + 7z(z - 1)^2 - (z - 1)$
   f. $x^4 - 4x$

Challenge Problems

4. Factor by grouping: $3ax + 12a + 2bx + 8b$

Summary For Unit 2

1. *A variable is a symbol or letter used to represent an unspecified value in expression.*
2. *An algebraic expression is a collection of variables and constant under algebraic operations of addition or subtraction. For example, $y + 10$ and $2t - 2 \times 8$ are algebraic expressions.*

The symbols used to show the four basic operations of addition, subtraction, multiplication and division are summarized in Table 2.1

Table 2.1		
Operation	Symbols	Translation
Addition	x + y	✓ Sum of x and y ✓ x plus y ✓ y added to x ✓ y more than x ✓ x increased by y ✓ the total of x and y
Subtraction	x – y	✓ difference of x and y ✓ x minus y ✓ y subtracted from x ✓ x decreased by y ✓ y less than x
Multiplication	$x \times y$, x(y), xy	✓ product of x and y ✓ x times y ✓ x multiplied by y
Division	$x \div y$, $\frac{x}{y}$, $^{x}/_{y}$	✓ Quotient of x and y ✓ x divided by y ✓ y divided into x ✓ ratio of x and y ✓ x over y

3. *For any rational numbers a, b and c*

   a. $a(b + c) = ab + ac$

   b. $a(b - c) = ab - ac$

   *These two properties are called the **distributive properties**.*

4. *If $(a + b)$ and $(c + d)$ are any two binomials whose product $(a + b)(c + d)$ is defined as $(a + b)(c + d) = ac + ad + bc + bd$.*
5. *The highest common factor (HCF) of numbers is the greatest number which is a common factor of the numbers.*

   The procedures of one of the ways to find the HCF is given below:

   1) List the factors of the numbers.

   2) Find the common factors of the numbers.

   3) Determine the highest common factors of these common factors.

Miscellaneous Exercise 2

I. State whether each statement is true or false for all positive integers x, y, z and w.

1. If a number y has z positive integer factors, then y and 2z integer factors.
2. If 2 is a factor of y and 3 is a factor of y, then 6 is a factor of y.
3. If y has exactly 2 positive integer factors, then y is a prime numbers.
4. If y has exactly 3 positive integer factors, then y is a square.
5. If y has exactly 4 positive integer factors, then y is a cube.
6. If x is a factor of y and y is a factor z, then x is a factor of z.

II. Choose the correct answer from the given four alternatives.

7. A triangle with sides 6, 8 and 10 has the same perimeter as a square with sides of length _______?

   a. 6 b. 4 c. 8 d. 12
8. If $x + y = 10$ and $x - y = 6$, what is the value of $x^3 - y^3$?

   a. 604 b. 504 c. 520 d. -520
9. If $ab + 5a + 3b + 15 = 24$ and $a + 3 = 6$, then $b + 5 =$ _____ ?

   a. 5 b. 50 c.4 d. 12
10. If $ab = 5$ and $a^2 + b^2 = 25$, then $(a + b)^2 =$ ________ ?

    a. 35 b. 20 c. 15 d. 30
11. If n is an integer, what is the sum of the next three consecutive even integers greater than 2n?

    a. 6n + 12 b. 6n + 10 c. 6n + 4 d. 6n + 8

12. One of the following equation is false.

a. $A = \frac{1}{2}bh$ for $h = \frac{2A}{b}$

c. $P = 2(\ell + w)$ for $\ell = \frac{p}{2} - w$

b. $A = 2s^2 + 4sh$ for $h = \frac{A - 2s^2}{4s}$

d. $A = \frac{1}{2}bh$ for $h = \sqrt{\frac{2A}{h}}$

13. If x = 6 and y = 2, then what is the value of $3x^2 - 4\left(2y - \frac{4}{12}\right) + 8$.

a. $\frac{304}{3}$ b. $\frac{-304}{3}$ c. $\frac{-348}{3}$ d. $\frac{108}{3}$

14. Find the value of y, if $y = x^2 - 6$ and x = 7.

a. 49 b. 7 c. 43 d. 45

15. If x = 2 and y = 3, then what is the value of $y^x + xy \times y + x$?

a. 9 b. -29 c. 29 d. 18

16. If a = 4 and b = 7, then what is the value of $\dfrac{a + \frac{a}{b}}{a - \frac{a}{b}}$?

a. 8 b. 1 c. $\frac{4}{3}$ d. $\frac{8}{3}$

III. Work out Problems

17. Simplify each of the following expressions.

a. $(x^3 + 2x - 3) - (x^2 - 2x + 4)$

b. $2x(3x + 4) - 3(x + 5)$

c. $x(y^2 + 5xy) + 2xy(3x - 2y)$

d. $2(a^2b^2 - 4a^3b^3) - 8(ab^2 - 3a^2b^2)$

18. Express the volume of this cube.

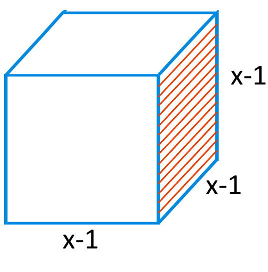


Figure 2.16 cube

19. Find the surface area of this cube.

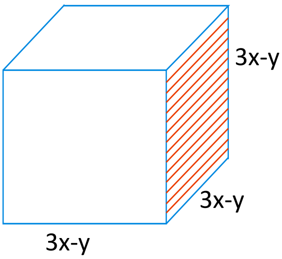


Figure 2.17 cube

20. Prove that the sum of five consecutive natural number is even.
21. Prove that 6(n + 6) – (2n + 3) is odd numbers for all $n \in \mathbb{N}$
22. Multiply the expressions.
    a. $(7x + y)(7x - y)$
    b. $(5k + 3t)(5k + 3t)$
    c. $(7x - 3y)(3x - 8y)$
    d. $(5z + 3)(z^2 + 4z - 1)$
    e. $\left(\frac{1}{3}m\text{-}n\right)^2$
    f. $(5a - 4b)(2a - b)$
    g. $\left(\frac{1}{5}x + 6\right)(5x - 3)$
    h. $(2h + 2\cdot7)(2h - 2\cdot7)$
    i. $(k - 3)^3$
    j. $(k + 3)^3$
23. Find the highest common factor for each expression.
    a. $12x^2 - 6x$
    b. $8x(x - 2) -2(x - 2)$
    c. $8(y + 5) + 9y(y + 5)$
    d. $y(5y + 1) - 8(5y + 1)$
    e. $4x(3x - y) + 5(3x - y)$
    f. $2(5x + 9) + 8x(5x + 9)$
    g. $8q^9 + 24q^3$
24. Find three consecutive numbers whose sum shall equal 45.
25. Find three consecutive numbers such that twice the greatest added to three times the least amount to 34.
26. Find two numbers whose sum is 36 and whose difference is 10.
27. If a is one factor of x, what is the other factor?
28. Find the value of (x+5) (x+2) + (x–3) (x –4) in its simplest form. What is the numerical value when x = -6?
29. Simplify (x + 2) (x + 10) – (x – 5) (x – 4). Find the numerical value of this expression when x = -3.

UNIT 3 LINEAR EQUATIONS AND INEQUALITIES

Unit outcomes

After Completing this unit, you Should be able to:

- understand the concept equations and inequalities.
- develop your skills on rearranging and solving linear equations and inequalities.
- apply the rule of transformation of equations and inequalities for solving problems.
- draw a line through the origin whose equation is given.

Introduction

In this unit you will expand the knowledge you already have on solving linear equations and inequalities by employing the very important properties known as the associative property and distributive property of multiplication over addition and apply these to solve problems from real life. More over you will learn how to set up a coordinate plane and drawing straight lines using their equation.

3.1 Further on Solutions of Linear Equations

Group Work 3.1

Discuss with your friends/partners.

1. Solve the following linear equations using equivalent transformation.
   a. $6x - 8 = 26$
   b. $14x + 6x = 64$
   c. $5x - 17 - 2x = 6x - 1 - x$
   d. $5x - 8 = -8 + 3x - x$
2. Solve the following linear equations.
   a. $7(x-1) - x = 3 - 5x + 3(4x - 3)$
   b. $0.60x + 3.6 = 0.40(x + 12)$
   c. $8(x + 2) + 4x + 3 = 5x + 4 + 5(x + 1)$
   d. $8y - (5y - 9) = -160$
   e. $3(x + 7) + (x - 8)6 = 600$
3. Do you recall the four basic transformation rules of linear equations. Explain.

3.1.1 Solution of Linear Equations Involving Brackets

Activity 3.1

Discuss with your friends/partners.

Solve the following linear equations.

a. $4(2x + 3) = 3(x + 8)$
b. $6(5x - 7) = 4(3x + 7)$
c. $4(8y + 3) = 6(7y + 5)$
d. $3(6t + 7) = 5(4t + 7)$
e. $7(9d - 5) = 12(5d - 6)$
f. $10x - (2x + 3) = 21$

To solve an equation containing brackets such as $5(4x + 6) = 50 - (2x + 10)$, you transform it into an equaivalent equation that does not have brackets. To do this it is necessary to remember the following rules.

Note: For rational numbers a, b and c,

a) $a + (b + c) = a + b + c$
b) $a - (b + c) = a - b - c$
c) $a(b + c) = ab + ac$
d) $a(b - c) = ab - ac$

Example 1: Solve: x – 2(x -1) = 1 – 4(x + 1) Using the above rules.

Solution x – 2 (x – 1) = 1 -4 (x +1) …Given equation

x – 2x + 2 = 1 – 4x – 4 ……. Removing brackets

x – 2x +4 x = 1 - 4 – 2………Collecting like terms

3x = -5 …………….Simplifying

$\frac{3x}{3} = \frac{-5}{3}$……….…….Dividing both sides by 3

x = $\frac{-5}{3}$…………..…. x is solved.

✓ Check: For x = $\frac{-5}{3}$

$$\frac{-5}{3} - 2\left(\frac{-5}{3} - 1\right) \stackrel{?}{=} 1 - 4\left(\frac{-5}{3} + 1\right)$$

$$\frac{-5}{3} + \frac{10}{3} + 2 \stackrel{?}{=} 1 + \frac{20}{3} - 4$$

$$\frac{-5}{3} + \frac{10}{3} + \frac{6}{3} \stackrel{?}{=} \frac{3}{3} + \frac{20}{3} - \frac{12}{3}$$

$$\frac{11}{3} = \frac{11}{3} \text{ (True)}$$

Example 2: Solve 4(x – 1) +3 (x + 2) = 5 (x – 4) using the above rules.

Solution 4(x -1) + 3(x + 2) = 5(x – 4) ……Given equation

4x- 4 + 3x + 6 = 5x - 20 ………Removing brackets

4x + 3x – 5x = -20 + 4 – 6…..…..Collecting like terms

2x = -22 ……………….Simplifying

$\frac{2x}{2} = \frac{-22}{2}$ ……………...Dividing both sides by 2

x = - 11 ………..……X is solved

✓ Check: For x = -11

$$4(-11 - 1) + 3(-11 + 2) \stackrel{?}{=} 5(-11 - 4)$$

$$4(-12) + 3(-9) \stackrel{?}{=} 5(-15)$$

$$-48 - 27 \stackrel{?}{=} -75$$

$$-75 = -75 \text{ (True)}$$

Exercise 3A

1. Solve each of the following equations, and check your answer in the original equations.

   a. $7x - 2x + 6 = 9x - 32$
   b. $21 - 6x = 10 - 4x$
   c. $2x - 16 = 16 - 2x$
   d. $8 - 4y = 10 - 10y$
   e. $8x + 4 = 3x - 4$
   f. $2x + 3 = 7x + 9$
   g. $5x - 17 = 2x + 4$
   h. $4x + 9 = 3x + 17$

2. Solve each of the following equations, and check your answer in the original equations.

   a. $7 - (x + 1) = 9 - (2x - 1)$
   b. $3y + 70 + 3(y - 1) = 2(2y + 6)$
   c. $5(1 - 2x) - 3(4 + 4x) = 0$
   d. $3 - 2(2x + 1) = x + 17$
   e. $4(8y + 3) = 6(7y + 5)$
   f. $8(2k - 6) = 5(3k - 7)$
   g. $5(2a + 1) + 3(3a - 4) = 4(3a - 6)$

Challenge problems

3. Solve the equation $8x + 10 - 2x = 12 + 6x - 2$.
4. solve the equation $-16(2x - 8) - (18x - 6) = -12 + 2(6x - 6)$.
5. Solve the equation $(8x - 4)(6x + 4) = (4x + 3)(12x - 1)$.
6. Solve for x in each of the following equations:

   a. $m(x + n) = n$
   b. $x(a + b) = b(c - x)$
   c. $mx = n(m + x)$

3.1.2 Solution of Linear Equations Involving Fractions

Group work 3.2

Discuss with your friends/partners.

1. **Work out**

   a. $\frac{2}{7} + \frac{3}{50}$
   b. $\frac{3}{8} + \frac{5}{8} + \frac{7}{8}$
   c. $2\frac{9}{10} + 1\frac{5}{8}$
   d. $3\frac{2}{5} + 2\frac{7}{15}$
   e. $1\frac{3}{4} + 2\frac{5}{16}$

2. Work out.

   a. $\frac{21}{4} - \frac{1}{15}$ b. $4\frac{7}{8} - 1\frac{2}{5}$ c. $6\frac{1}{5} - 5\frac{1}{7}$ d. $7\frac{4}{7} - 4\frac{2}{5}$

3. Work out.

   a. $\frac{2}{35} \times 2\frac{5}{6}$ b. $2\frac{1}{3} \times \frac{7}{10}$ c. $21\frac{1}{7} \times 1\frac{3}{5}$ d. $3\frac{5}{6} \times 2\frac{5}{7}$

4. Work out

   a. $3\frac{5}{9} \div \frac{20}{9}$ b. $36\frac{7}{3} \div 2\frac{2}{5}$ c. $4\frac{3}{5} \div \frac{2}{3}$ d. $2\frac{3}{2} \div \frac{15}{2}$

5. In a school, $\frac{7}{16}$ of the students are girls. What fraction of the students are boys?

6. A box containing tomatoes has a total weight of $5\frac{7}{8}$ kg.
   The empty box has a weight of $1\frac{1}{4}$kg. what is the weight of the tomatoes?

7. A machine takes $5\frac{1}{2}$ minutes to produce a special type of container. How long would the machine take to produce 15 container?

From grade 5 and 6 mathematics lesson you have learnt about addition, subtraction, multiplication and division of fractions. All of these are shown on the following discussion.

Adding fractions

It is easy to add fractions when the denominators (bottom) are the same:

Easy to add:

$$\frac{35}{29} + \frac{39}{29} = \frac{74}{29}$$

Denominators are the same

what about this?

$$\frac{38}{9} + \frac{37}{11} = ?$$

Denominators are different

Adding fractions with the same denominator

$$\frac{7}{20} + \frac{2}{20} = \frac{9}{20}$$

Add the numerators (top)

Add them over the same denominators, (bottom)

Example 3:

Adding fractions with different denominators

$$\frac{38}{9}+\frac{37}{11}=?$$

First find equivalent fractions to these ones which have the same denominator (bottom):

Fractions equivalent to $\frac{38}{9}$

$$\frac{38\times 11}{9\times 11}=\boxed{\frac{418}{99}}$$

Fractions equivalent to $\frac{37}{11}$

$$\frac{37\times 9}{11\times 9}=\boxed{\frac{333}{99}}$$

$$\frac{418}{99}+\frac{333}{99}=\frac{751}{99}$$

Equivalent to $\frac{38}{9}$ Equivalent to $\frac{37}{11}$

So $\frac{38}{9}+\frac{37}{11}=\frac{751}{99}$

Note: To add fractions, find equivalent fractions that have the same denominator or (bottom).

Subtracting fractions

It is easy to subtract fractions when the denominators (bottom) are the same:

Easy to subtract:

$$\frac{7}{12}-\frac{26}{12}=\frac{-19}{12}$$

Denomintaors are the same

What about this?

$$\frac{5}{9}-\frac{1}{4}=?$$

Denominators are different.

Example 4: (Subtracting fraction with different denominators)

work out $\frac{5}{9}-\frac{1}{4}$

Solution: Find equivalent fractions to these ones which have the same denominator (bottom). An easy way is to change both denominator to 36 because $9\times 4 = 36$ is LCM of the denominators.

$$\frac{5}{9} \xrightarrow{\times 4} \frac{20}{36} \qquad \frac{1}{4} \xrightarrow{\times 9} \frac{9}{36}$$

$$\frac{20}{36} - \frac{9}{36} = \frac{11}{36}$$

Equivalent to $\frac{5}{9}$ Equivalent to $\frac{1}{4}$

So $\frac{5}{9} - \frac{1}{4} = \frac{11}{36}$

Note: To subtract fractions, find equivalent fractions that have the same denominator (bottom).

Multiplying fractions

To multiply two fractions, multiply the numerators together and multiply the denominators together.

For example,

$$\frac{50}{18} \times \frac{7}{10} = \frac{350}{180}$$

Multiply the numerators (top)

Multiply the denominators (bottom)

You can simplify this to $\frac{35}{18}$ (by dividing the top and bottom of $\frac{350}{180}$ by 10).

Therefore, $\frac{50}{18} \times \frac{7}{10} = \frac{35}{18}$

Dividing fractions

To divide fractions, invert or take the reciprocal of the dividing fraction (turn it upside down) and multiply by the divisor.

For example

Chang the "÷ " $\frac{1}{21} \div \frac{3}{7} = ?$

Sign in to a "×" sign — change the fraction you are dividing by up side down. This is called **inverting** the fraction.

$\frac{1}{21} \times \frac{7}{3} = \frac{1}{9}$

Now let us consider linear equations having fractional coefficients.

Example 5: Solve $\frac{x+1}{3} + \frac{x-1}{10} = 12.$

Solution: $\frac{x+1}{3} + \frac{x-1}{10} = 12$ …..Given equation

The LCM of the denominators is 3 × 10 = 30 since 3 and 10 do not have any common factors.

Therefore, multiplying both sides by 30.

$$30\left(\frac{x+1}{3} + \frac{x-1}{10}\right) = 30\times12$$

$$30\left(\frac{x+1}{3}\right) + 30\left(\frac{x-1}{10}\right) = 30\times12 \text{……..By the distributive property}$$

$$10(x + 1) + 3(x - 1) = 360 \text{ ………Simplifying}$$

$$10x + 10 + 3x - 3 = 360 \text{ ………Removing brackets}$$

$$13x + 7 = 360 \text{ ………Collecting like terms}$$

$$13x + 7 - 7 = 360 - 7 \text{……Subtracting 7 from both sides}$$

$$13x = 353 \text{………Simplifying}$$

$$\frac{13x}{13} = \frac{353}{13} \text{ … … ….Dividing both sides by 13}$$

$$x = \frac{353}{13}$$

The solution set is $\left\{\frac{353}{13}\right\}$.

✓ Check: $\frac{x+1}{3} + \frac{x-1}{10} = 12$

$$\frac{\frac{353}{13}+1}{3} + \frac{\frac{353}{13}-1}{10} \overset{?}{=} 12$$

$$\frac{353+13}{39} + \frac{353-13}{130} \overset{?}{=} 12$$

$$\frac{366}{39} + \frac{340}{130} \overset{?}{=} 12$$

$$\frac{47580+13260}{5070} \stackrel{?}{=} 12$$

$$\frac{60840}{5070} \stackrel{?}{=} 12$$

12 = 12 (True)

Example 6: Solve $\frac{7}{24} = \frac{x}{8} + \frac{1}{6}$.

Solution: The LCM of the denominators is 24.

$24\left(\frac{7}{24}\right) = 24\left(\frac{x}{8} + \frac{1}{6}\right)$ ….Multiplying both sides by 24.

$24\left(\frac{7}{24}\right) = 24\left(\left(\frac{x}{8}\right) + 24\left(\frac{1}{6}\right)\right)$…Distributive property

7 = 3x + 4 …………Removing brackets

7 – 4 = 3x + 4 – 4………Subtracting 4 from both sides

3 = 3x …………Simplifying

Or $\frac{3x}{3} = \frac{3}{3}$… … ..Dividing both sides by 3

$x = \frac{3}{3} = 1$

The solution set is {1}.

Example 7: Solve $\frac{x+1}{2} + \frac{x+2}{3} + \frac{x+3}{4} = 16$.

Solution: The LCM of the denominators is 12.

$12\left(\frac{x+1}{2} + \frac{x+2}{3} + \frac{x+3}{4}\right) = (12 \times 16)$…Multiplying both sides by 12.

$12\left(\frac{x+1}{2}\right) + 12\left(\frac{x+2}{3}\right) + 12\left(\frac{x+3}{4}\right) = 12\times16$……Distributive property

6(x + 1) + 4(x +2) +3(x + 3) = 12×16…….Simplifying

6x +6 + 4x + 8 + 3x + 9 = 192………Removing brackets

13x + 23 -23 = 192-23...Subtracting 23 from both sides

13x = 169 …………Simplifying

$\frac{13x}{13} = \frac{169}{13}$……….Dividing both sides by 13

x = 13

The solution set is {13}.

✓ **Check:** $\frac{13+1}{2} + \frac{13+2}{3} + \frac{13+3}{4} \stackrel{?}{=} 16$

$$\frac{14}{2} + \frac{15}{3} + \frac{16}{4} \stackrel{?}{=} 16$$

$$7 + 5 + 4 \stackrel{?}{=} 16$$

16 = 16 (True)

Example 8: Solve $\frac{1}{3}(x + 7) - \frac{1}{2}(x + 1) = 4$.

Solution: The LCM of the denominators 3 and 2 is 6.

$\frac{1}{3}(x + 7) - \frac{1}{2}(x + 1) = 4$ ………Given equation

$6\left[\frac{1}{3}(x + 7) - \frac{1}{2}(x + 1)\right] = 6\times4$ ……Multiply both sides by 6.

$6\left[\frac{1}{3}(x + 7)\right] - 6\left[\frac{1}{2}(x + 1)\right] = 6\times4$ …Distributive property

$2(x + 7) - 3(x + 1) = 24$ …Simplifying

$2x + 14 - 3x - 3 = 24$ …Removing brackets

$2x - 3x + 14 - 3 = 24$

$-x + 11 = 24$ ….Collecting like terms

$-x + 11 - 11 = 24 - 11$...Subtracting 11 from both sides

$-x = 13$ ……………….Simplifying

$\frac{-x}{-1} = \frac{13}{-1}$………..Dividing both sides by -1.

$x = -13$

The solution set is $\{-13\}$.

✓ Check: $\frac{1}{3}(x + 7) - \frac{1}{2}(x + 1) \stackrel{?}{=} 4$

$\frac{1}{3}(-13 + 7) - \frac{1}{2}(-13 + 1) \stackrel{?}{=} 4$

$\frac{1}{3}(-6) - \frac{1}{2}(-12) \stackrel{?}{=} 4$

$-2 + 6 \stackrel{?}{=} 4$

$4 = 4$ (True)

Exercise 3B

1. Solve each of the following equations.

a. $\frac{x}{10} = \frac{2}{3}$

b. $\frac{6n}{2} - \frac{3n}{2} = 3\frac{1}{2}$

c. $\frac{-x}{2} + 6 = -3\frac{2}{8}$

d. $\frac{-3}{5} + \frac{x}{10} = \frac{-1}{5} - \frac{x}{5}$

e. $\frac{2x}{5} - \frac{2}{3} = \frac{x}{2} + 6$

f. $\frac{3x}{7} + \frac{35x}{8} = 10$

g. $\frac{5x}{13} + \frac{5x}{26} = 1$

h. $\frac{12}{23} - x = 4$

2. Solve each of the following equations and check your answer in each case by inserting the solution in original equation.

a. $\frac{5x}{6}+\frac{2}{3}=\frac{-1x}{6}-\frac{5}{3}$

b. $\frac{3}{7}x-\frac{1}{4}=\frac{-4x}{7}-\frac{5}{4}$

c. $\frac{24}{5}w+14=62-\frac{6}{10}w$

d. $\frac{9x}{7}-10=\frac{48x}{7}+14$

e. $\frac{2x+2}{2}+\frac{3x+6}{3}+\frac{4x+16}{4}=-6$

f. $\frac{4+2x}{6x}=\frac{12}{5x}+\frac{2}{15}$

g. $\frac{2x+7}{3}-\frac{x-9}{2}=\frac{5}{2}$

h. $\frac{2x+3}{6}-\frac{x-5}{4}=\frac{3}{8}$

Challenge Problems

3. Solve the following equations.

a. $12-\frac{x-2}{2}=\frac{6-x}{4}+\frac{x-4}{4}$

b. $\frac{2x-10}{11}-\frac{2x-4}{7}=10x-17\frac{1}{2}$

c. $\frac{x+9}{4}-\frac{x-12}{5}=6^{2}$

d. $0.78-\frac{1}{25}h=\frac{3}{5}h-0.5$

3.1.3 Solve Word Problems Using Linear Equations

Group work 3.3

1. **Two complementary angles are drawn such that one angle is 10° more than seven times the other angle. Find the measure of each angle.**
2. **A certain number of two digits is three times the sum of its digits, and if 45 be added to it the digits will be reversed; find the number.**

Mathematical problems can be expressed in different ways. Common ways of expressing mathematical problems are verbal or words and formulas or open statements. In this sub-unit you will learn how to translate verbal problems to formulas or mathematical expressions so that you can solve it easily. It is important to translate world problems to open statements because it will be clear and concise.

Although there is no one definite procedure which will insure success to translate word problems to open statements to solve it, the following steps will help to develop the skill.

Table 3.1 Problem – solving Flow chart for word problems

Step 1	Read the problem carefully.	• Familiarize yourself with the problem. Identify the unknown, and if possible estimate the answer.
Step 2	Assign lables to unknown quantities	• Identify the unknown quantity or quantities. Let x represent one of the unknowns. Draw a picture and write down relevant formulas.
Step 3	Develop a verbal models	• Write an equation in words.
Step 4	Write a mathematical equation	• Replace the verbal model with a mathematical equation using x or other variable.
Step 5	Solve the equation	• Solve for the variable using steps for solving linear equations.
Step 6	Interpet the results and write the final answer in words	• Once you have obtained a numerical value for the variable, recall what it represents in the context of the problems.
Step 7	Check all answers back into the original statements of the problems	

Example 9: The sum of a number and negative ten is negative fifteen. Find the number.

Solution:

Let x represent the unknown number — Step 1: Read the problem; Step 2: Label the unknown

$(\text{a number}) + (-10) = -15$ — Step 3: Develop a verbal model

$x + (-10) = -15$ — Step 4: Write the equation

$x + (-10) + 10 = -15 + 10$ — Step 5: Solve for x

$x = -5$

Therefore, the number is -5. — Step 6: Write the final answer in words.

Example 10: **(Applications involving sales Tax)**

A video game is purchased for a total of Birr 48.15 including sales tax. If the tax rate is 7%. Find the original price of the video game before sales tax is added.

Solution: Step 1: Read the problem

Let x represent the price of the video game.

0.07x represents the amount of sales tax. Step 2: Label variables

$\begin{pmatrix}\text{orignal}\\ \text{price}\end{pmatrix} + \begin{pmatrix}\text{sales}\\ \text{tax}\end{pmatrix} = \begin{pmatrix}\text{total}\\ \text{cost}\end{pmatrix}$ Step 3: Write a verbal equation

x + 0.07x = Birr 48.15 Step 4: Write a mathematical equation

1.07x = 48 .15 Step 5: Solve for x multiply by 100 to clear decimals

100(1.07x) = 100(48.15)

107x = 4815

$\frac{107x}{107} = \frac{4815}{107}$ Step 6: Divide both sides by 107

$x = \frac{4815}{107}$

x = 45 Step 7: Interpret the results and write the answer in words.

Therefore, the original price was Birr 45.

Example 11: **(Applications involving consecutive integers)**

Find three consecutive even numbers which add 792.

Solution: Step 1: Read the problem

Let the smallest even number be x. Step 2: label the unknow

Then the other even numbers are (x+2) and (x+4) Step 3: Develop a verbal model

Because they are consecutive even numbers.

x+(x+2) +(x+4) = 792 Step 4: Write the equation

3x + 6 = 792

3x = 786

x = 262

The three even numbers are 262, 264 and 266. Step 5: Write the final answer in Word

✓ **Check**

262 + 264 +266 = 792 Step 6: Check

Example 12: **(Applications involving ages)**

The sum of the ages of a man and his wife is 96 years. The man is 6 years older than his wife. How old is his wife?

Solution: let m = the man's age and w = the wife's age.

$\Rightarrow$ m + w= 96 …………….Translated equation (1)

$\Rightarrow$ m = 6 + w ……………...Translated equation (2)

$\Rightarrow$ (6 + w) + w = 96 ……... Substituting equation (2) into 1

$\Rightarrow$ 2w + 6 = 96 ……………Collecting like terms

$\Rightarrow$ 2w = 96 – 6 ……………Subtracting 6

$\Rightarrow$ 2w = 90 ………………..Collecting like terms

$\Rightarrow$ w = 45 …………………Divided both sides by 2

Therefore, the age of his wife is 45 years old.

Exercise 3C

Solve each problem by forming an equation.

1. The sum of three consecutive numbers is 276. Find the numbers.
2. The sum of three consecutive odd number is 177. Find the numbers.
3. Find three consecutive even numbers which add 1524.
4. When a number is doubled and then added to 13, the result is 38. Find the number.
5. Two angles of an isosceles triangle are x and (x+10). Find two possible values of x.
6. A man is 32 years older than his son. Ten years ago he was three times as old as his son. Find the present age of each.

Challenge Problems

7. A shop –keeper buys 20kg of sugar at Birr y per kg. He sells 16kgs at Birr $\left(y + \frac{3}{4}\right)$ per kg and the rest at Birr (y + 1) per kg. what is his profit.
8. A grocer buys x kg of potatoes at Birr 1.50 per kg and y kg of onions at Birr 2.25 per kg. how much money does he pay in Birr?

9. If P is the smallest of four consecutive even integers, what is their sum interms of P?
10. The sum of a certain number and a second number is -42. The first number minus the second number is 52. Find the numbers.

3.2 Further on Linear Inequalities

Activity 3.2

Discuss with your friends/partners.

1. Can you recall the definition of linear inequality?
2. Discuss the four rules of transformation of linear inequalities using examples and discuss the result with your teacher.
3. Solve the following linear inequalities.

a. $4x - 16 < 12, x \in \mathbb{W}$

b. $\frac{2}{3}x < -4(x - 5), x \in \mathbb{Z}^+$

c. $20 - \frac{3}{2}x \geq \frac{3}{2}x - 18, x \in \mathbb{Q}^+$

d. $0.5(x - 8) \leq 10 + \frac{3}{2}x, x \in \mathbb{Q}$

From grade 6 and 7 mathematics lesson you have learnt about to solve linear inequalities in one variable based on the given domain.

Example 13: (Solving an inequality)

Solve the inequality $-3x + 8 \leq 22$.

Solution:

$-3x + 8 \leq 22$…………Given inequalities

$-3x + 8 - 8 \leq 22 - 8$ ……..Subtracting 8 from both sides

$-3x \leq 14$ ……Simplifying

$\frac{-3x}{-3} \geq \frac{14}{-3}$…Dividing both sides by -3; reverse the inequality sign

$x \geq \frac{-14}{3}$…….Simplifying

Therefore, the solution set is $\left\{x : x \geq \frac{-14}{3}\right\}$.

Example 14: (Solving an inequality)

Solve the inequality $24x - 3 < 4x + 10, x \in \mathbb{Q}$.

Solution:

$24x - 3 < 4x + 10\ x \in \mathbb{Q}$ …………Given inequalities

$24x - 3 + 3 < (4x + 10) + 3$ ……….Adding 3 from both sides

$24x < 4x + 13$…………………Simplifying

24x – 4x < 4x – 4x +13……..Subtracting 4x from both sides

20x < 13……………..Simplifying

$\frac{20x}{20} < \frac{13}{20}$……..Dividing both sides by 20

x $< \frac{13}{20}$……….Simplify

Therefore, the solution set is $\left\{x: x < \frac{13}{20}\right\}$.

Example 15: (**Solving an inequality**)

Solve the inequality 4x -6 > 10, x∈ ℕ.

Solution: 4x – 6 > 10 …………Given inequalities

(4x -6) + 6 >10 + 6 ……..Adding 6 from both sides

4x > 16 ………..Simplifying

$\frac{4x}{4} > \frac{16}{4}$…………..Dividing both sides by 4

x > 4

The solution of the inequality is x > 4.

Therefore, the solution set is $\{x: x > 4\} = \{5, 6, 7, 8, 9, \ldots\}$.

Example 16: (**Solving an inequality**)

Solve the inequality $\frac{-1}{4}x + \frac{1}{6} \leq 2 + \frac{2}{3}x$, x∈ℤ.

Solution: $\frac{-1}{4}x + \frac{1}{6} \leq 2 + \frac{2}{3}x$………Given inequalities

$12\left(\frac{-1}{4}x + \frac{1}{6}\right) \leq 12\left(2 + \frac{2x}{3}\right)$……Multiply both sides by 12 to clear fractions.

$12\left(\frac{-1}{4}x\right) + 12\left(\frac{1}{6}\right) \leq 12(2) + 12\left(\frac{2}{3}x\right)$…….Apply the distributive property

-3x + 2 ≤ 24 + 8x …………Simplifying

-3x – 8x + 2 ≤ 24 + 8x -8x ……..Subtracting 8x from both sides

-11x + 2 ≤ 24 ………..Collecting like terms

-11x + 2 – 2 ≤ 24 – 2 ………Subtracting 2 from both sides

-11x ≤ 22 ……………Simplifying

$\frac{-11x}{-11} \geq \frac{22}{-11}$ …………Dividing both sides by -11. Reverse the inequality sign.

x ≥ -2

There fore, the solution set is {-2, -1, 0, 1, 2, 3, …}.

Example 17: (**Solving an inequality**)

Solve the inequality $3x - 2(2x - 7) \leq 2(3 + x) - 4$, $x \in \mathbb{N}$.

Solution:

$3x - 2(2x - 7) \leq 2(3 + x) - 4$ …Given inequalities

$3x - 4x + 14 \leq 6 + 2x - 4$ ………Removing brackets

$-x + 14 \leq 2x + 2$ ………………...Simplifying

$-x - 2x + 14 \leq 2x - 2x + 2$ ………Subtracting 2x from both sides

$-3x + 14 \leq 2$ ……………………..Simplifying.

$-3x + 14 - 14 \leq 2 - 14$…………...Subtracting 14 from both sides

$-3x \leq -12$ ……………………….Simplifying

$\frac{-3x}{-3} \geq \frac{-12}{-3}$ ……………Dividing both sides by -3. Reverse the inequality sign

$x \geq 4$

The solution of the inequality is $x \geq 4$.

Therefore, the solution set is $\{x: x \geq 4\} = \{4, 5, 6, 7, 8, 9, \ldots\}$.

Exercise 3D

1. Solve the following inequalities:
   a. $\frac{1}{2}(x + 4) \geq \frac{3}{4}(x - 2)$
   b. $\frac{x}{4} + 5 \leq x + 4$
   c. $8x - 5 > 13 - x$
   d. $4x + 6 > 3x + 3$
   e. $\frac{1}{4}x + 7 \leq \frac{1}{3}x - 2$
   f. $9 + \frac{1}{3}x \geq 4 - \frac{1}{2}x$
   g. $\frac{1}{2}(2x + 3) > 0$
2. Solve each of the following linear inequality in the given domain.
   a. $4 - \frac{5}{6}x > \frac{3}{2}x - 8$, $x \in \mathbb{Q}$
   b. $4y - 6 < \frac{1}{2}(28 - 2y)$, $y \in \mathbb{W}$
   c. $\frac{5}{3}x < -8(x - 6)$, $x \in \mathbb{Z}^+$
   d. $-2(12 - 2x) \geq 3x - 24$, $x \in \mathbb{Q}^+$
   e. $5x + 6 \leq 3x + 20$, $x \in \mathbb{N}$
   f. $\frac{3y}{4} + \frac{1}{6} > \frac{17}{10}$, $y \in \mathbb{Z}$
   g. $6x \geq 16 + 2x - 4$, $x \in \mathbb{Z}$
   h. $10x + 12 \leq 6x + 40$, $x \in \mathbb{N}$
3. Eight times a number increased by 4 times the number is less than 36. What is the number?
4. If five times a whole number increased by 3 is less than 13, then find the solution set.

Challenge Problems

5. Solve each of the following linear inequalities:
   a. $3(x + 2) - (2x - 7) \leq (5x - 1) - 2(x + 6)$
   b. $6 - 8(y + 3) + 5y > 5y - (2y - 5) + 13$
   c. $-2 - \frac{W}{4} \leq \frac{1+w}{3}$
   d. $-0.703 < 0.122x - 2.472$
   e. $3.88 - 1.335t \geq 5.66$

3.3 Cartesian Coordinate System

3.3.1 The Four Quadrants of the Cartesian Coordinate Plane

Group work 3.4

1. Write down the coordinates of all the points marked red in Figure 3.1 to the right .

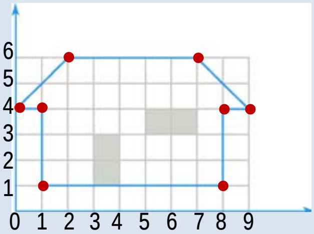


Figure 3.1

2. Write the coordinates of the points A, B, C, D, E, F, G and H shown in Figures 3.2 to the right.

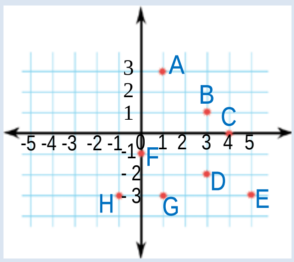


Figure 3.2

3. Name the quadrant in which the point of p(x, y) lines when:
   a. $x > 0, y > 0$
   b. $x < 0, y > 0$
   c. $x > 0, y < 0$
   d. $x < 0, y < 0$

For determing the position of a point on a plane you have to draw two mutually perpendicular number lines. The horizontal line is called the **X –axis**, while the vertical line is called the **Y-axis**. These two axes together set up a plane called the **Cartesian coordinate planes**. The point of intersection of these two axis is called the **origin**. On a suitably chosen scale, points representing numbers on the X-axis are called **X-coordinates or abscissa**, while chose on the y-axis are called **Y-coordinates or ordinat**. The x-coordinate to the right of the y-axis are positive, while those to the left are negative. The y- coordinates above and below the X-axis are positive and negative respectively. Let XOX' and YOY' be the **X-axis and the Y-axis** respectively and let P be any point in the given plane. For determing the coordinates of the point P, you draw lines through P parallel to the coordinate axis, meeting the X-axis in M and the y-axis in N.

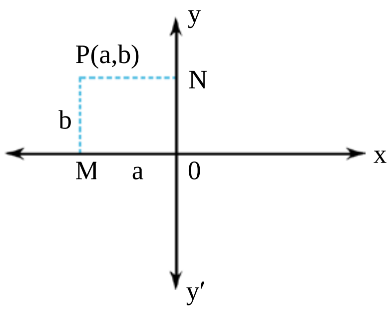


Figure 3.3

The two axes divide the given plane into four quadrants. Starting from the positive direction of the X-axis and moving the anticlockwise (counter clockwise) direction, the quadrants which you come across are called **the first**, **the second**, **the third** and **the fourth quadrants** respectively.

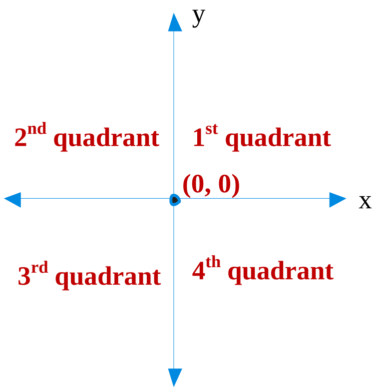


Figure 3.4

Note: i. In the first quadrant all points have a positive abscissa and a positive ordinate.
ii. In the second quadrant all points have a negative abscissa and a positive ordinate.
iii. In the third quadrant all points have a negative abscissa and a negative ordinate.
iv. In the fourth quadrant all points have a positive abscissa and a negative ordinate.

EXERCISE 3E

1. Draw a pair of coordinate axes, and plot the point associated with each of the following ordered pair of numbers.

A(-3, 4)	D (0, -3)	G (0, 6)
B(4, 6)	E (-3, -2)	H (2.5, 3)
C(4, -3)	F (-5, 6)	I (-2, 4.5)

2. Based on the given Figure 3.5 to the right answer the following questions.
   a. Write the coordinates of the point A, B, P, S, N and T.
   b. Which point has the coordinates (-1, -2)?
   c. Which coordinate of the points Q is zero?
   d. Which coordinate of the points D and M is the same?
   e. To which axis is the line DM parallel?
   f. To which axis is the line AT parallel?
   g. If F is any point on the line AT, state its y-coordinate.
   h. To which axis is the line PQ parallel?

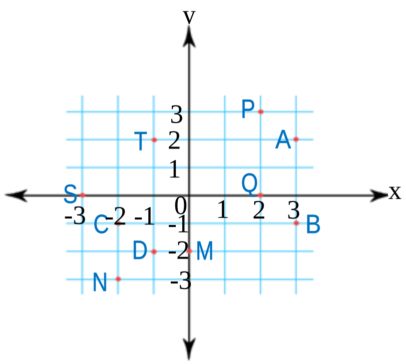


Figure 3.5

Challenge Problems

3. Answer the following:

   a. On which axis does the point A(0,6) lie?

   b. In which quadrant does the point B(-3, -6) lie?

   c. Write the coordinates of the point of intersection of the x-axis and y- axis.

3.3.2 Coordinates and Straight Lines

Group Work 3.5

1. Write down the equations of the lines marked (a) to (d) in the given Figure 3.6 to the right.

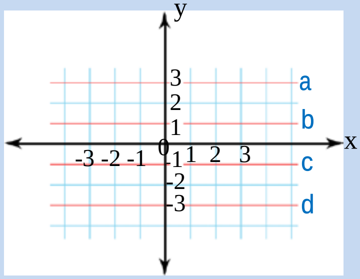


Figure 3.6

2. Write down the equations of the lines marked (a) to (d) in the given Figure 3.7 to the right.

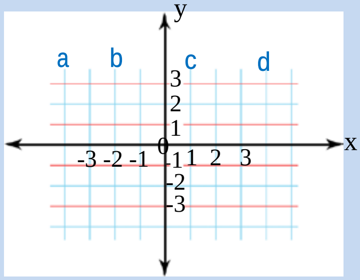


Figure 3.7

3. Draw the graphs of the following equations on the same coordinate system:

a. $y = x$ **c. $y = 4x$**

b. $y = -x$ **d. $y = -4x$**

4. True or false. If the statement is false, rewrite it to be true.

a. The line $x = 30$ is horizontal.   **b. The line $y = -24$ is horizontal.**

5. True or false. If the statement is false, rewrite it to be true.

a. A line parallel to the y – axis is vertical.

b. A line perpendicular to the x – axis is vertical.

For exercise 6 – 9, identify the equation as representing a vertical line or a horizontal line.

6. $2x + 7 = 10$ **7. $9 = 3 + 4y$**   **8. $-3y + 2 = 9$**   **9. $7 = -2x – 5$**

10. Write an equation representing the x – axis.

11. write an equation representing the y – axis.

Graph of an equation of the form x = a ($a \in \mathbb{Q}$)

The graph of the equation x= a ($a \in \mathbb{Q}$, $a \neq 0$) is a line parallel to the y-axis and at a distance of **a unit** from it.

Note: i. If a > 0, then the line lies to the right of the y-axis.
ii. If a <0, then the line lies to the left of the y-axis.
iii. The graph of the equation x= 0 is the y-axis.

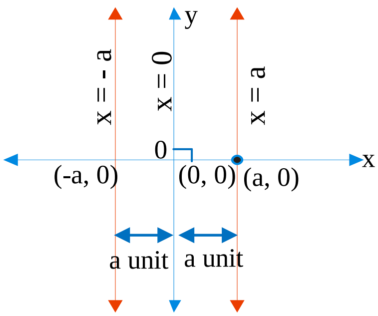


Figure 3.8

Example 18: Draw the graphs of the following straight lines.

a. x = 6 b. x = -6

Solution: First by drawing tables of values for x, and y in which x-is constant and following this you plot these points and realize that the points lie vertical line.

a..

x	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
y	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8

b.

x	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
y	-5	-4	-3	-2	-1	0	1	2	3	4	5

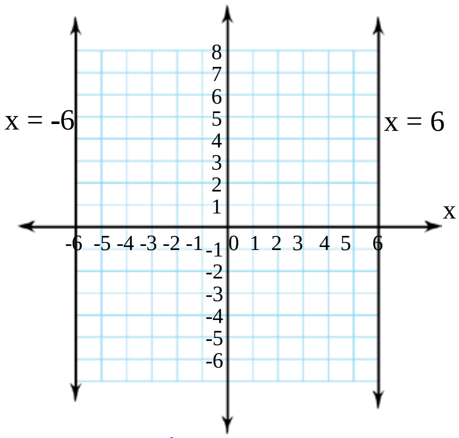


Figure 3.9

Graph of an equation of the form y = b (b ∈ ℚ)

The graph of the equation y= b (b∈ ℚ, b≠0) is the line parallel to the x-axis and at a distance of b from it.

Note: i. If b >0, then the line lies above the x-axis.
ii. If b < 0, then the line lies below the x-axis.
iii. The graph of the equation y= 0 is the x-axis.

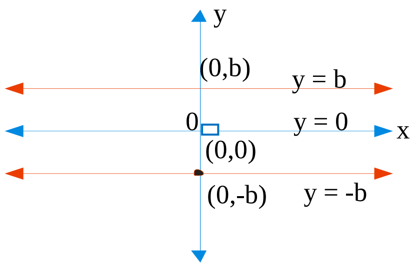


Figure 3.10

Example 19: Draw the graphs of y = 4.

Solution: First by drawing tables of values for x, and y in which y is constant and following this you plot these points and realize that the points lie horizontal line.

x	-4	-3	-2	-1	0	1	2	3	4
y	4	4	4	4	4	4	4	4	4

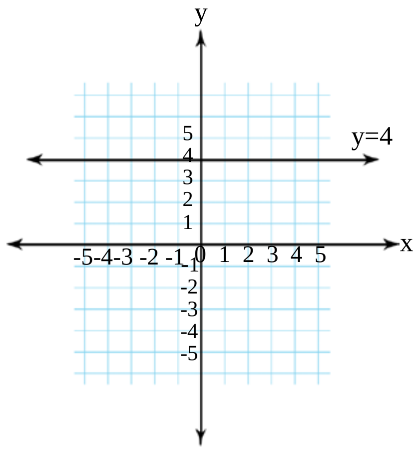


Figure 3.11

Graph of an equation of the form y = mx (m∈ ℚ and m ≠0)

In grade 6 and 7 mathematics lesson we discussed about y = kx, where y is directly proportional to x, with constant of proportionality k. For example y = 4x where y is directly proportional to x with constant of poroportionality 4. Similarly how to draw the graph of y= mx, (m∈ℚ), look at the following examples.

Example 20: Draw the graphs of y = 5x.

Solution:

Step i: Choose some values for x, for example let x= -2,-1, 0, 1 and 2.

Step ii: Put these values of x into the equation y = 5x:

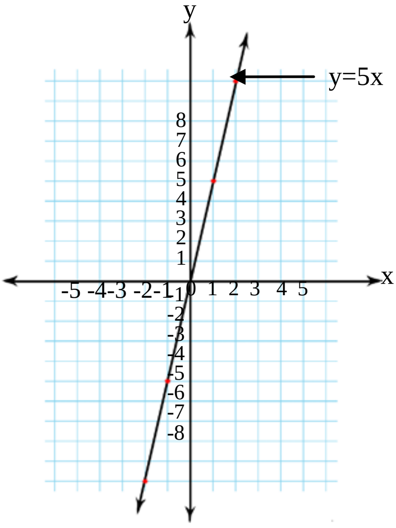
When x= -2: y =5(-2) = -10

When x = -1: y = 5(-1) = -5

When x = 0: y = 5(0) = 0

When x = 1: y = 5(1) = 5

When x = 2: y = 5(2) = 10



Step iii: Write these pairs of values in a table.

x	-2	-1	0	1	2
y	-10	-5	0	5	10

Step iv: Plot the points (-2, -10), (-1, -5), (0, 0) (1, 5) and (2, 10) and join them to get a straight line.

Figure 3.12

Step v: Lable the line y = 5x.

Example 21: Draw the graphs of y = -5x.

Solution:

Step i: Choose some values for x, for example let x= -2, -1, 0, 1 and 2.

Step ii: Put these values of x into the equation y= -5x

When x= -2: y = -5(-2) =10

When x = -1: y= -5(-1) = 5

When x = 0: y = -5(0)=0

When x= 2: y = -5(2) = -10

Step iii: Write these pairs of values in a table.

x	-2	-1	0	1	2
y	10	5	0	-5	-10

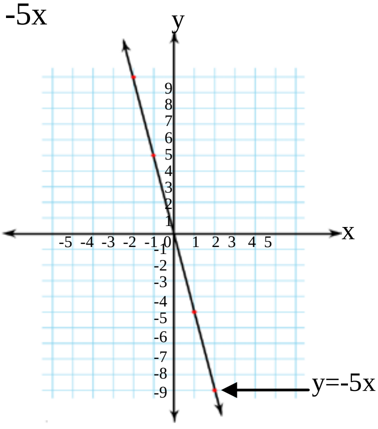


Figure 3.13

Step iv: Plot the points (-2, 10), (-1, 5), (0, 0) (1, -5) and (2, -10) and join them to get a straight line.

Step v: Label the line y=-5x

3.3.3. The Slope "m" Of Straight Line

Activity 3.3

Discuss with your friends.

1. **What is a slope?**
2. **What is the slope of a line parallel to the y-axis?**
3. **What is the slope of a horizontal line?**
4. **What is the slope of a line parallel to the x – axis?**
5. **What is the slope of a line that rises from left to right?**
6. **What is the slope of a line that falls from left to right?**
7. **a. Draw a line with a negative slope.**
   b. Draw a line with a positive slope.
   c. Draw a line with an undefined slope.
   d. Draw a line with a slope of zero.

From your every day experience, you might be familiar with the idea of **slope**. In this sub – topic you learnt how to calculate the slope of a line by dividing the change in the y – value by change in the x – value, where the y – value is the vertical height gained or lost and the x – value is the horizontal distance travelled.

$$\textbf{Slope} = \frac{\text{change in y} - \text{value}}{\text{change in x} - \text{value}}$$

In Figure 3.14 to the right, consider a line drawn through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$. From P to Q the change in the x coordinate is $(x_2 - x_1)$ and the change in the y coordinate is $(y_2 - y_1)$. By definition, the slope of the line AB is given by:

$$\frac{y_2 - y_1}{x_2 - x_1}; x_2 \neq x_2$$

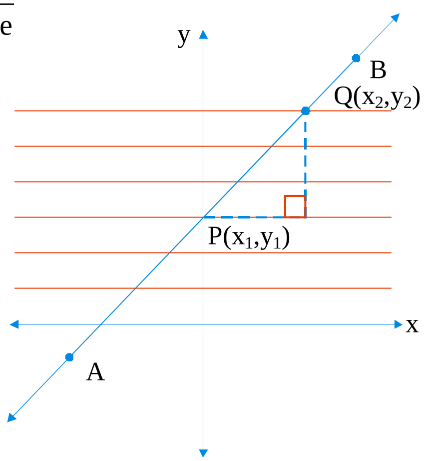


Figure 3.14

Note: If we denote the slope of a line by the letter "m".

Definition 3.1: If $x_1 \neq x_2$ the slope of the line through the points (x_1, y_1) and (x_2, y_2) is the ratio:

$$\text{Slope} = m = \frac{\text{Change in } y - \text{value}}{\text{change in } x - \text{value}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Group work 3.6

Discuss with your friends (partners).

1. In Figure 3.15 below, determine the slope of the roof.

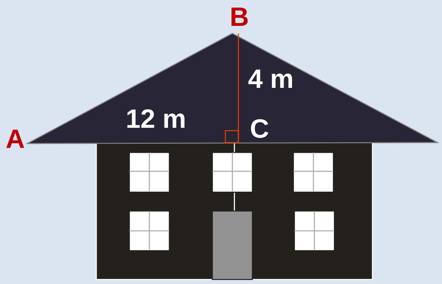


Figure 3.15

2. State the slope of the straight line that contains the points p(1, -1) and Q(8, 10).

3. Find the slope of a line segment through points (-7, 2) and (8,6).

4. Find the slope of each line.

a) y = 4 **b) x = 7**

Example 22: Find the slope of the line passing through the point P(-4, 2) and Q(8, -4).

Solution: $\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-2}{8-(-4)} = \frac{-6}{12} = \frac{-1}{2}$

Therefore, $-\frac{1}{2}$ is the coefficient of x in the line equation $y = -\frac{1}{2}x$.

Example 23: Find the slope of the line passing through each of the following pairs of points.

a) P(4, -6) and Q(10, -6) b) P $\left(\frac{-1}{4},-4\right)$ and Q$\left(\frac{-1}{4},4\right)$

Solution:

a. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-(-6)}{10-4} = \frac{-6+6}{6} = \frac{0}{6} = 0$

b. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-4)}{\frac{-1}{4}-\left(\frac{-1}{4}\right)} = \frac{8}{0}$ undefined

Note: **i. The horizontal line has a slope of 0.**
ii. The vertical line has no slope (not defined).

Example 24: Draw the graphs of the following equations on the same Cartesian coordinate plane.

a. $y = \frac{7}{6}x$ b. y = -3x c. y = 4x d. $y = \frac{2}{3}x$

Solution: First to draw the graph of the equation to calculated some ordered pairs that belongs to each equation shown in the table below.

x	-3	-2	-1	0	1	2	3
$y=\frac{7}{6}x$	$\frac{-7}{2}$	$\frac{-7}{3}$	$\frac{-7}{6}$	0	$\frac{7}{6}$	$\frac{7}{3}$	$\frac{7}{2}$
y = -3x	9	6	3	0	-3	-6	-9
y = 4x	-12	-8	-4	0	4	8	12
$y=\frac{2}{3}x$	-2	$\frac{-4}{3}$	$\frac{-2}{3}$	0	$\frac{2}{3}$	$\frac{4}{3}$	2

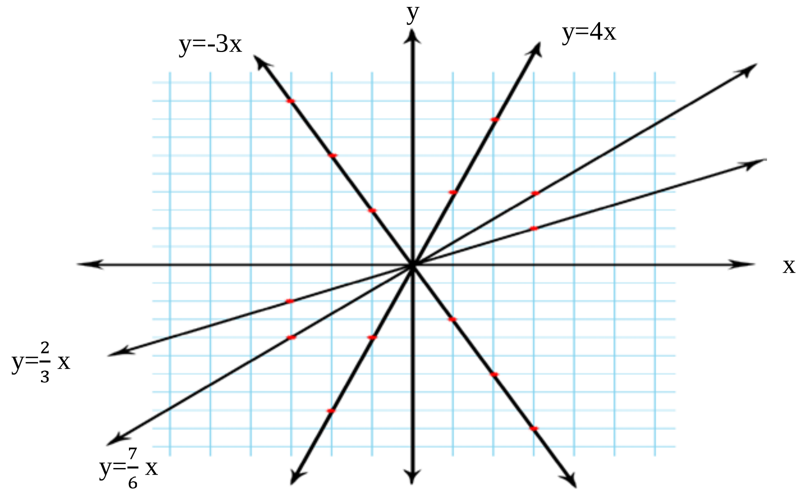


Figure 3.16

From the above graphs, you can generalize that:

i. All orderd pairs, that satisfy each linear equation of the form $y = mx$ $(m \in \mathbb{Q}, m \neq 0)$ lies on a straight lines that pass through the origin.

ii. The equation of the line $y = mx$, m is called the **slope** of the line, and the graph passes the **1st and 3rd quadrants if m > 0**, and the graph passes through the **2nd and 4th quadrants if m< 0.**

EXERCISE 3F

1. Draw the graphs of the following equations on the same coordinate system:

   a. $y = -6x$ b. $y = 6x$ c. $y = \frac{5}{2}x$ d. $y = \frac{-5}{2}x$

2. Draw the graphs of the following equations on the same coordinate system:

   a. $y + 4x = 0$ c. $x = 3$ e. $2x – y = 0$

   b. $2y = 5x$ d. $x + 4 = 0$ f. $\frac{3}{2}x - \frac{y}{2} = 0$

3. Complete the following tables for drawing the graph of $y = \frac{2x}{3}$

x	1	6	3
y			
(x, y)			

Challenge Problems

4. Point (3, 2) lies on the line $ax+2y = 10$. Find a .
5. Point (m, 5) lies on the line given by the equation $5x – y = 20$. Find m.
6. Draw and complete a table of values for the graphs $y=2x -1$ and $y= x- 2$
7. a. Show that the choice of an ordered pair to use as (x_1, y_1) does not affect the slope of the line through (2, 3) and (-3, 5).

   b. Show that $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$

   For Exercise 8 – 11 fined the slope of the line that passes through the two points.

8. $P\left(\frac{-2}{7}, \frac{1}{3}\right)$ and $Q\left(\frac{8}{7}, \frac{-5}{6}\right)$
9. $A\left(\frac{1}{2}, \frac{3}{5}\right)$ and $B\left(\frac{1}{4}, \frac{-4}{5}\right)$
10. C (0, 24) and D (30, 0)

11. $E\left(0, \frac{5}{7}\right)$ and $F\left(0, \frac{9}{26}\right)$
12. Find the slope between the points
    A (a + b, 4m – n) and B (a – b, m + 2n)
13. Find the slope between the points
    C(3c – d, s + t) and D (c – 2d, s – t)
14. Write the equation of the line which has the given slope "m" and which passes through the given point.
    a. (2, 10) and m = - 4 b. (4, -4) and m = $\frac{3}{2}$ c. (0, 0) and m = $\frac{3}{5}$
15. State the slope and y-intercept of the line 2x + y + 1 = 0.
16. Find the slope and y-intercept of $y - y_o = m(x - x_o)$ where x_o and y_o are constants.
17. Find the slope and y-intercept of each line:
    a. (x + 2) (x + 3) = (x – 2) (x – 3) + y
    b. x = mu + b
18. State the slope and y-intercept of each linear equations.
    a. 6(x + y) = 3(x – y)
    b. 2(x + y) = 5(y + 1)
    c. 5x + 10y – 20 = 0
19. Write the slope-intercept equation of the line that passes through (2,5) and (-1,3).

Summary For unit 3

1. *You can transform an equation into an equivalent equation that does not have brackets. To do this it is necessary to remember the following rules.*

   a. $a + (b + c) = a + b + c$ **c.** $a\,(b + c) = ab + ac$

   b. $a - (b + c) = a - b - c$ **d.** $a\,(b - c) = ab - ac$

2. *The following rules are used to transform a given equation to an equivalent equation.*

   a. *For all rational numbers a, b and c:*

   If $a = b$ *then* $a + c = b + c$ *and* $a - c = b - c$*. that is, the same number may be added to both sides and the same number maybe subtracted from both sides without affecting the equality.*

   b. *For all rational numbers a, b and c where* $c \neq 0$*:*

   If $a = b$ *then* $ac = bc$ *and* $\frac{a}{c} = \frac{b}{c}$*. That is both sides may be multiplied by the same non-zero number and both sides may be divided by the same non-zero number without affecting the equality.*

3. *To solve word problems, the following steps will help you to develop the skill. The steps are:*

   a. *Read the problem carefully, and make certain that you understand the meanings of all words.*

   b. *Read the problem a second time to get an overview of the situation being described and to determine the known facts as well as what is to be found.*

   c. *Sketch any figure, diagram or chart (if any) that might be helpful in analyzing the problem.*

   d. *Choose a variable to represent an unknown quantity in the problem.*

   e. *Form an equation containing the variable which translates the conditions of the problem.*

   f. *Solve the equation.*

   g. *Check all answer back into the original statements of the problem.*

4. *The following rules are used to transform a given inequality to an equivalent inequality.*

   a. *For all rational numbers a ,b and c, if* $a < b$ *then* $a + c < b + c$ *or* $a - c < b - c$*. That is, if the same number is added to or subtracted from both sides of an inequality, the direction of the inequality remains unchanged.*

b. *For all rational numbers a, b and c*

i. *If* $a < b$ ***and*** $c > 0$, *then* $ac < bc$ ***or*** $\frac{a}{c} < \frac{b}{c}$. *That is, if both sides of an inequality are multiplied or divided by the same positive number, the direction of the inequality is unchanged.*

ii. *If* $a < b$ ***and*** $c < 0$, *then* $ac > bc$ ***or*** $\frac{a}{c} > \frac{b}{c}$. *That is, if both sides are multipled or divided by the same negative number, the direction of the inequality is* ***reversed***.

5. *The two axes divide the given plane into four quadrants. Starting from the positive direction of the X-axis and moving the anticlockwise direction, the quadrants which you come across are called* ***the first***, ***the second***, ***the third*** *and the* ***fourth*** *quadrants respectively.*

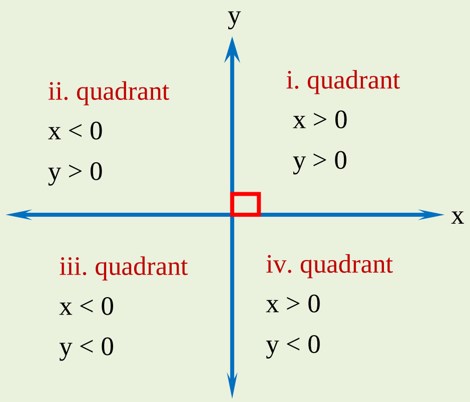


Figure 3.17

6. *If* $x_1 \neq x_2$ *the slope of the line through the points* (x_1, y_1) *and* (x_2, y_2) *is the ratio:*

$$\textbf{Slope} = m = \frac{\text{Change in } y\text{-value}}{\text{Change in } x\text{-value}}$$

$$= \frac{Y_2 - Y_1}{X_2 - X_1}$$

7. *All orderd pairs, that satisfy each linear equation of the form*

$y = mx$ $(m \in \mathbb{Q}, m \neq 0)$ *lies on a straight lines that pass through the origin.*

8. *The equation of the line* $y = mx$, m *is called the slope of the line, and the graph passes the 1st and 3rd quadrants if* $m > 0$, *and the graph passes through the* 2^{nd} *and* 4^{th} *quadrants if* $m < 0$.

Miscellaneous Exercise 3

I. Write true for the correct statements and false for the incorrect ones.

1. For any rational numbers a, b and c, then $a(b + c) = ab + ac$.
2. If any rational number $a > 0$, $ax + b > 0$, then the solution set is $\left\{x: x \geq \frac{-b}{a}\right\}$.
3. If any rational number $a < 0$, $ax + b > 0$, then the solution set is $\left\{x: x < \frac{-b}{a}\right\}$.
4. The equation of the line $y = 4x$, 4 is the slope of the line and the graph passes the 2nd and 3rd quadrants, since $4 > 0$.
5. The equation of the line $y= 4x + 6$ that pass through the origin of coordinates.
6. The graphs of the equation $y= b$ ($b \in \mathbb{Q}$, $b \neq 0$), if $b > 0$ then the equation of the line lies above the x-axis.
7. The graph of the equation $x= a$ ($a \in \mathbb{Q}$, $a \neq 0$), if $a < 0$ then the equation of the lines to right of the y-axis.

II. Choose the correct answer from the given alternatives

8. In one of the following linear equations does pass through the origin?

   a. $y = \frac{3}{7}x + 10$ c. $y = \frac{5}{8}x$

   b. $y = -3x + \frac{3}{5}$ d. $y = 2x - 6$

9. The solution set of the equation $\frac{3x+2}{5} - \frac{2x-5}{3} = 2$ is:

   a. $\{1\}$ b. $\{-1\}$ c. $\left\{\frac{1}{2}\right\}$ d. $\left\{\frac{3}{2}\right\}$

10. The solution set of the equation $2x + 3(5 - 3x) = 7(5 - 3)$ is:

    a. $\{5\}$ b. $\frac{1}{7}$ c. $\{3\}$ d. $\left\{\frac{5}{3}\right\}$

11. If $\frac{2}{5x} = 2 + \frac{1}{x}$, ($x \neq 0$), then which of the following is the correct value of x?

    a. $\frac{1}{8}$ b. $\frac{3}{10}$ c. $\frac{-7}{10}$ d. $\frac{-3}{10}$

12. If x is a natural number, then what is the solution set of the inequality $0.2x - \frac{1}{5} \leq 0.1x$?

    a. $\{x: x \leq 0 \text{ or } x \geq 1\}$ b. ϕ c. $\{1, 2\}$ d. $\{0, 1, 2\}$

13. Which one of the following equations has no solution in the set of integers $\mathbb{Z}$?
    a. $6x + 4 = 10$
    b. $8x + 2 = 4x - 6$
    c. $9 - 12x = 3$
    d. $\frac{3}{2}x - 3 = 3x$
14. What is the solution set of the inequality $20(4x - 6) \leq 80$ in the set of positive integers?
    a. $\{1, 2, 3, 4\}$ b. $\{1, 2\}$ c. ϕ d. $\{0, 1, 2, 3, 4\}$
15. The sum of the ages of a boy and his sister is 32 years. The boy is 6 years older than his sister. How old is his sister?
    a. 15 b. 19 c. 14 d. 13

III. Work out problems

16. Solve each of the following linear equation by the rules of transformation.
    a. $4x + 36 = 86 - 8x$
    b. $12x - 8 + 2x - 17 = 3x - 4 - 8 + 74$
    c. $4(2x - 10) = 70 + 6x$
    d. $20 - 2x = 62(x - 3)$
    e. $2(6y - 18) - 102 = 78 - 18(y + 2)$
    f. $7(x + 26 + 2x) = 5(x + 7)$
17. Sove each of the following equations.
    a. $\frac{2x+7}{3} - \frac{x-9}{2} = \frac{5}{2}$ b. $\frac{x+3}{6} - \frac{x-5}{4} = \frac{3}{8}$ c. $\frac{x+2}{3} + \frac{x+3}{8} = \frac{5}{6}$
18. Solve each of the following linear inequalities by the rules of transformation.
    a. $6x - 2 < 22$
    b. $-9 \leq 3x + 12$
    c. $8x - 44 < 12(x - 7)$
    d. $8(x - 3) \geq 15x - 10$
    e. $\frac{x}{5} - 8 > \frac{-x}{3}$
    f. $6(2 + 6x) \geq 10x - 12$
19. **(word problems)**
    a. The sum of three consecutive odd integers is 129. Find the integers.
    b. Two of the angles in a triangle are complementary. The third angle is twice the measure of one of the complementany angles. What is the measure of each of the angles?
    c. Abebe is 12 years old and his sister Aster is 2 years old. In how many years will Abebe be exactly twice as old as Aster?
20. Draw the graphs of the equations $y = \frac{8}{3}x$ and $y = -\frac{8}{3}x$ on the same coordinate plane. Name their point of intersection as p. State the coordinate of the point p.

21. Find the equation of the line with y-intercept (0,8) and slope $\frac{3}{5}$.
22. Find the slope and y-intercept of $y = 10x - \frac{1}{3}$.
23. Find the slopes of the lines containing these points.
    a) (4,-3) and (6, -4)
    b) $\left(\frac{1}{8},\frac{1}{4}\right)$ and $\left(\frac{3}{4},\frac{1}{2}\right)$
    c) $\left(\frac{1}{2},\frac{1}{4}\right)$ and $\left(\frac{3}{2},\frac{3}{4}\right)$
24. Find the slope of the line x = -24.
25. Find a and b, if the points P(6,0) and Q (3,2) lie on the graph of ax + by = 12.
26. Points P(3,0) and Q(-3,4) are on the line ax + by = 6. Find the values of a and b.
27. Point (a,a) lies on the graph of the equation 3y = 2x – 4. Find the value of a.
28. Find anequation of the line containing (3,-4) and having slope -2. If this line contains the points (a,8) and (5,b), find a and b.

UNIT 4 SIMILAR FIGURES

Unit outcomes

After Completing this unit, you should be able to:

- know the concept of similar figures and related terminologies.
- understand the condition for triangles to be similar.
- apply tests to check whether two given triangles are similar or not.

Introduction

You may see the map of Ethiopia either in smaller or larger size, but have you asked yourself about the difference and likness of these maps? In geometery this concept is described by "similarity of plane figures" and you learn this concept here in this unit. You begin this by studying similarity of triangles and how to compare their areas and perimeters.

4.1 Similar Plane Figures

Activity 4.1

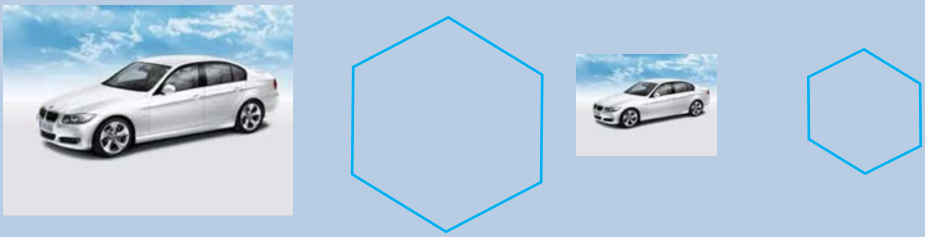
Discuss with your teacher

1. Which of the following maps are similar?

(a) (b) (c) (d)

Figure 4.1

2. Which of the following pictures or polygons are similar?



(a) (b) (c) (d)

Figure 4.2

3. Which of the following polygons are similar?

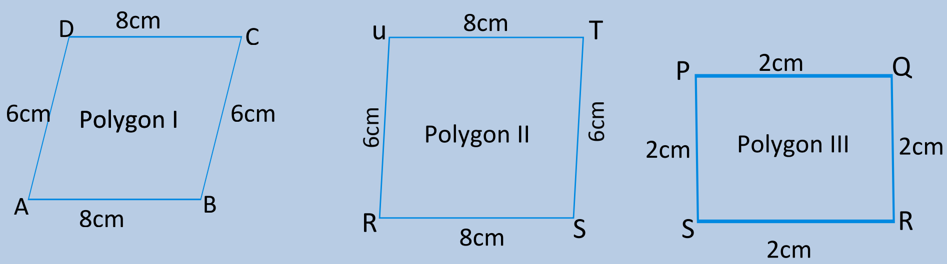
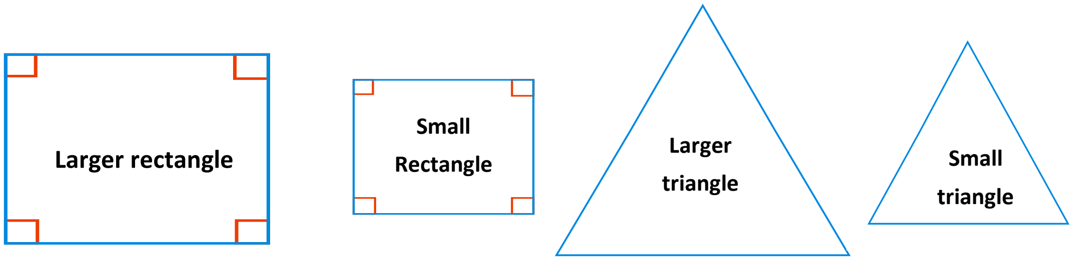


Figure 4.3

Similar geometric Figures are figures which have exactly the same shape. See Figure 4.4, each pair of figures are similar.



(a) (b)

Figure. 4.4 Geometric Figures

Therefore, geometric figures having the same shape, equal corresponding angles and corresponding sides are proportional are called **similar figures**.

4.1.1 Illustration and Definition of Similar Figures

Group Work 4.1

1. A square and a rectangle have corresponding angles congruent.

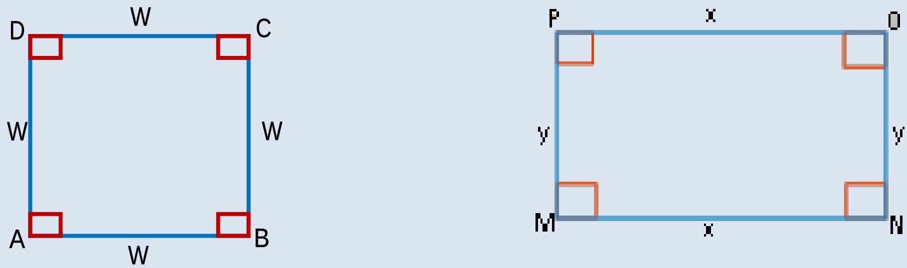
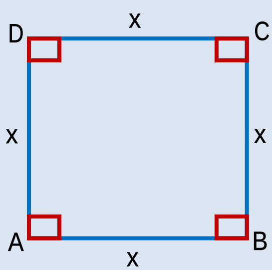
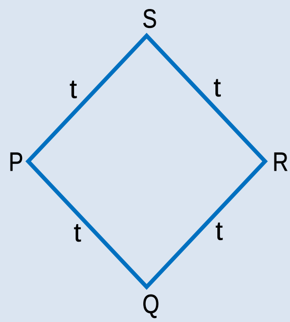


Figure. 4.5

Are they similar? Why?

2. A square and a rhombus have the corresponding sides proportional.





Are they similar? Why?

Figure. 4.6

3. Which members of these families of shape are similar:
   a) squares d) circles g) regular hexagons
   b) rectangles e) equilateral triangles h) trapeziums
   c) Parallelograms f) isosceles triangles

4. Which members of these families of solid shape are similar:
   a) cubes c) spheres e) pyramids
   b) cuboids d) tetrahedrons f) cones

Definition 4.1: **Two polygons are similar, if:**

i. their corresponding sides are proportional.

ii. their corresponding angles are congruent (equal).

Example1: Which of the following polygons are similar? Which are not? state the reason.

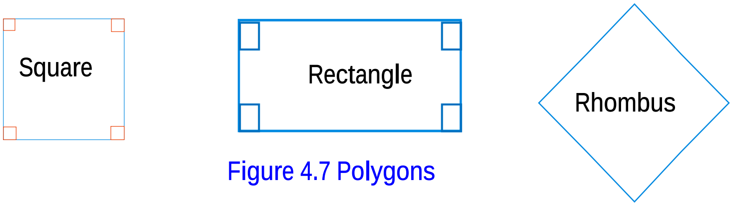
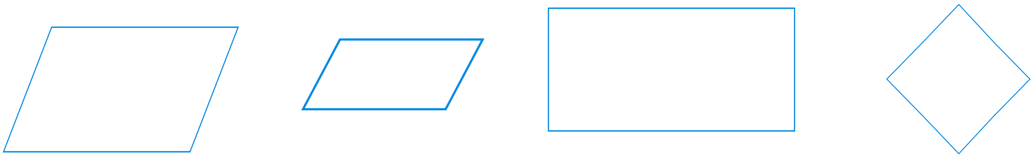


Figure 4.7 Polygons

Solution: From the given Figure 4.7 above

a. The square and the rectangle are not similar because their corresponding angles are congruent but their corresponding sides are not proportional.
b. The square and the rhombus are not similar because their corresponding sides are proportional but their corresponding angles are not congruent.
c. The rectangle and the rhombus are not similar because their corresponding angles are not congruent and corresponding sides are not proportional.

Example2: Tell whether each pairs of in Figures 4.8 is similar or not.



(a) (b)

Figure 4.8 Polygons

Solution:

a. Paris of the quadrilaterals have the same shape and angles but have not the same size. Therefore, they are not similar.
b. Paris of the quadrilaterals have not the same shape. Therefore, they are not similar.

Exercise 4A

Which of the following figures are always similar?

a. Any two circles.
b. Any two line segments.
c. Any two quadrilaterals.
d. Any two isosceles triangles.
e. Any two squares.
f. Any two rectangles.
g. Any two equilateral triangles.

4.1.2 Scale Factors and Proportionality

Activity 4.2

Discuss with your friends.

1. Have you observed what they do in the film studio?
2. How do you see films in the cinema house?
3. How are the pictures enlarged on the cinema screen?
4. What is meant by scale factor?
5. What is meant by proportional sides of similar figures?

To discuss Activity 4.2, it is important to study central enlargement (central stretching). **Central enlargement** which is either increases or decreases the size of figures with out affecting their shapes.

Under an enlargement

1. Lines and their images are parallel.
2. Angles remain the same.
3. All lengths are increased or decreased in the same ratio.

Positive enlargement

In Figure 4.9 triangle $A_1B_1C_1$ is the image of triangle ABC under enlargement. O is the centre of enlargement and the lines AA_1, CC_1 and BB_1 when produced must all pass through O.

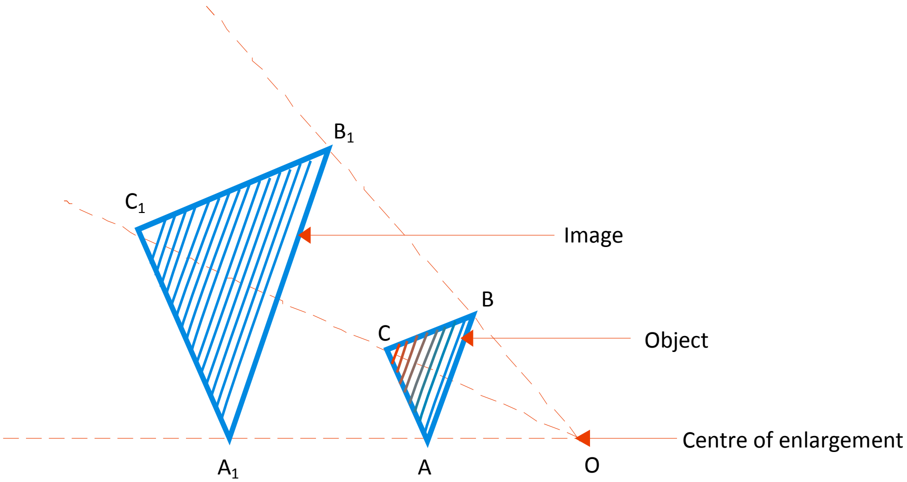


Figure 4.9 Positive enlargement

The enlargement in Figure 4.9 is called a **positive enlargement,** because both the object and its image are on the same side of the centre of enlargement. Also the image is further from O than the object.

Note: - AB is parallel to A_1B_1.
- BC is parallel to B_1C_1.
- AC is parallel to A_1C_1.
- Angle BAC≅ angle $B_1 A_1 C_1$.
- Angle ABC ≅ angle $A_1 B_1 C_1$.
- Angle BCA≅angle $B_1C_1 A_1$.

? "In Figure 4.9 above the object and the image are similar why?"

Example3: Given the rectangle, ABCD and A as the centre of enlargement. Draw the image AB'C'D' after enlargement of each side of ABCD twice.

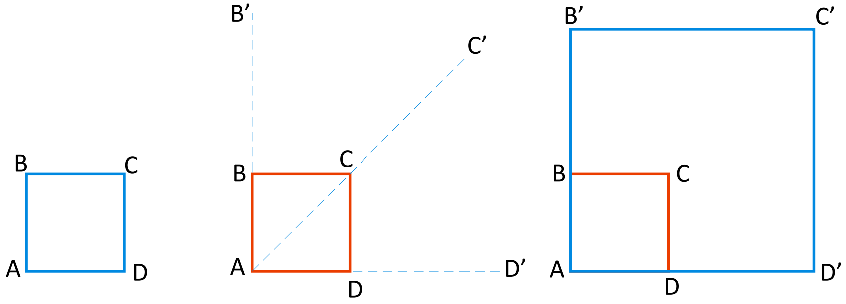


Figure. 4.10

Solution:

Point A is the centre of enlargement and is fixed. **A' is at A**. Since each sides of A'B'C'D' enlarged twice of each side of ABCD,

$\frac{AB}{A'B'} = \frac{AD}{A'D'} = \frac{AC}{A'C'} = \frac{DC}{D'C'} = \frac{1}{2}$ or A'B'=2AB, A'D'=2AD A'C'=2AC and, D'C' = 2DC. The number 2 in this equation is called **the constant of proportionality or scale factor**.

? "Can you define a scale factor based on example 3 above?"

Definition 4.2: Scale factor – the ratio of corresponding sides usually expressed numerically so that:

$$\text{Scale factor} = \frac{\text{length of line segment on the enlargement}}{\text{length of line segment on the original}}$$

Notation of scale factor = K

Example 4: The vertices of triangle ABC have co-ordinates A(2,1), B(4,1) and C(3,4). Find the co-ordinates of triangle $A_1B_1C_1$ after an enlargement, scale factor 2, with centre at O.

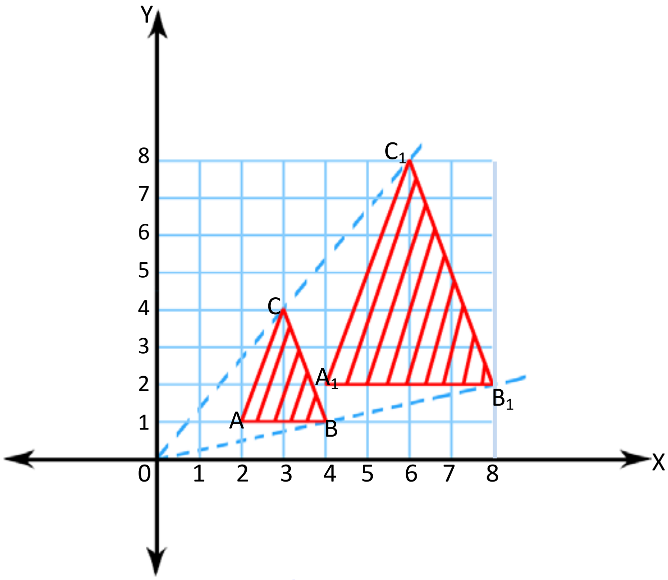


Figure. 4.11

Solution:

In Figure 4.11 you can see the object, triangle ABC and its image under enlargement, triangle $A_1B_1C_1$ with co-ordinates: A_1 (4,2), B_1 (8,2) C_1 (6,8).

Example 5: Give the shape PQRS and point O. Draw the image P'Q'R'S' after enlargement of each side of PQRS twice.

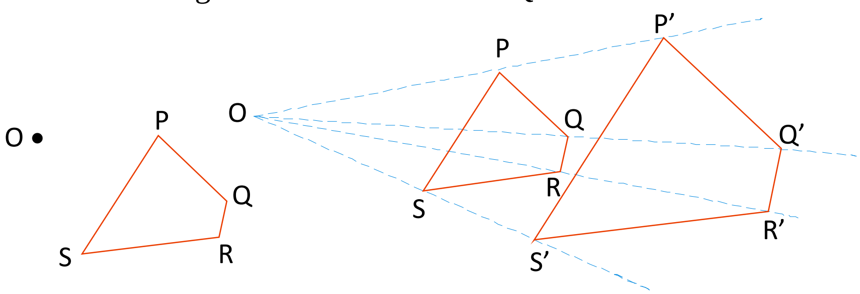


Figure 4.12

Solution: Join O to P, Q, R and S. Since each sides of P' Q' R' S' enlarged twice of each side of PQRS.

$$\frac{OP}{OP'} = \frac{OQ}{OQ'} = \frac{OR}{OR'} = \frac{OS}{OS'} = \frac{1}{2} \quad \text{or}$$

OP'=2OP, OQ'= 2OQ, OR'=2OR and OS'=2OS

Hence, the number 2 is called **the constant of proportionality** or **scale factor.**

? Can you define a central enlargement (central stretching) in your own word?

Definition 4.3: **A mapping which transform a figure following the steps given below is called *central enlargement.***

Step i: Mark any point O. This point O is called center of the central enlargement.

Step ii: Fix a number K. This number K is the constant of proportionality.

Step iii: Determine the image of each point A such as A' such that A'O=KAO.

Step iv: The image of point O is itself.

According to the definition 4.3 when you find the image of a plane figure,

i. If K > 1, the image figure is larger than the object figure.

ii. If 0 < K < 1, the image figure is smaller than the object figure.

iii. If K=1, the image figure is congruent to the object figure.

Example 6: Enlarge triangle PQR by scale factor 3 and O is the centre of enlargement as shown below.

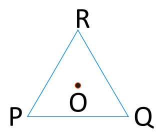


Figure 4.13

Solution:

Copy triangle PQR and the point O inside the triangle.

Step i: Make point P', R' and Q' on $\overrightarrow{OP}, \overrightarrow{OQ}$ and $\overrightarrow{OR}$ such that $P'O = 3PO$, $Q'O = 3QO$ and $R'O = 3RO$ see Figure 4.14 to the right.

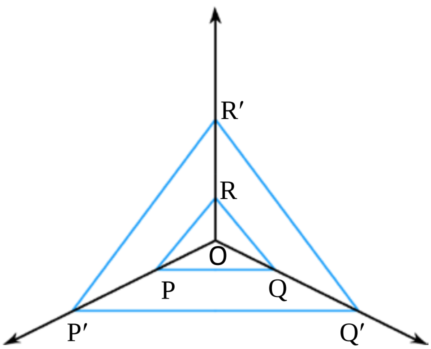


Figure 4.14

Stepii: Join the points P',Q', R' with line segment to obtain $\triangle P'Q'R'$ (which is the required Figure).

Exercise 4B

1. Draw the image of the shape KLMN after an enlargement by scale factor $\frac{1}{2}$ with center O. Label the image K' L' M' N'.

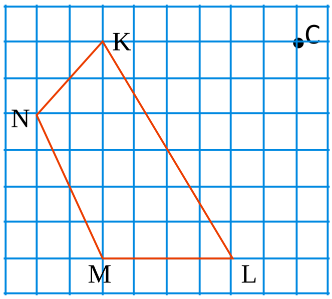


Figure 4.15

2. Work out the scale factor of the enlargement that takes in Figure 4.16, triangle ABC on the triangle LMN.

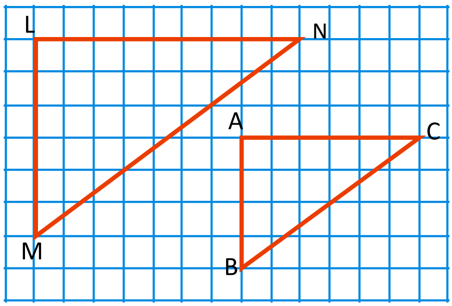


Figure 4.16

3. Copy the Figure 4.17 below. With O as centre, draw the image of the shaded shape after enlargement by:

   a. scale factor $\frac{1}{4}$

   b. scale factor $\frac{3}{4}$

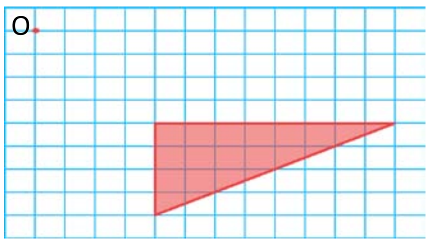


Figure 4.17

4.2 Similar Triangles

4.2.1 Introduction to Similar Triangles

Group Work 4.2

1. Consider Figure 4.18 below:

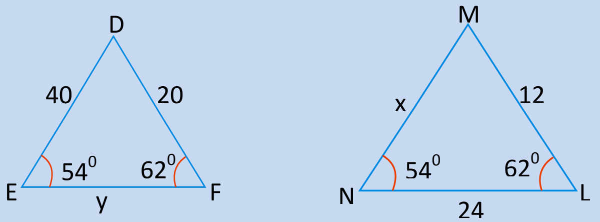


Figure 4.18

a. Are ΔDEF similar to ΔMNL? Why?

b. If ΔDEF similar to ΔMNL then find the value of X and Y.

2. Consider Figure 4.19 below:

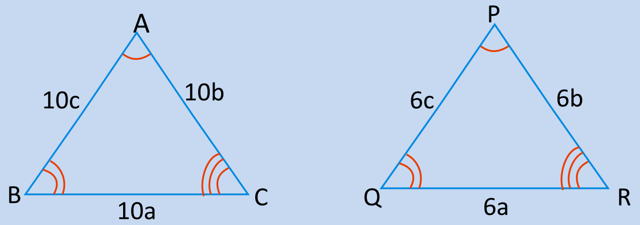


Figure 4.19

a. ΔABC is similar to ΔPQR. Explain the reason.

b. ΔABC is not similar to ΔPRQ. Explain the reason.

3. ΔXYZ is given such that ΔDEF similar ΔXYZ. Find XY and YZ when the scale factor from ΔXYZ to ΔDEF is 6 and DE=7, EF=12 and XZ=36.

You have define similar polygon in section 4.1.1. You also know that any polygon could be dived into triangles by drawing the diagonals of the polygon. Thus the definition you gave for similar polygons could be used to define similar triangles.

Definition 4.4: ΔABC is similar to ΔDEF, if

i. their corresponding sides are proportional.

ii. their corresponding angles are congruent.

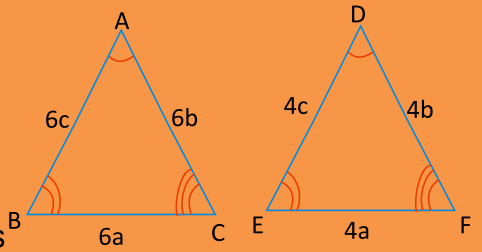


Figure. 4.20

That is symbolically:

$\Delta ABC \sim \Delta DEF$ if and only if

1. $\angle A \equiv \angle D$
2. $\angle B \equiv \angle E$
3. $\angle C \equiv \angle F$

corresponding angles are congruent

4. $\dfrac{AB}{DE} = \dfrac{BC}{EF}$
5. $\dfrac{BC}{EF} = \dfrac{AC}{DF}$
6. $\dfrac{AB}{DE} = \dfrac{AC}{DF}$

corresponding sides are proportional

or the above three facts 4, 5 and 6 can be summarized as

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = K$$

Note: From definition (4.4) of similar triangles it is obvious that similarity is a transitive relation. That is: If $\Delta ABC \sim \Delta DEF$ and $\Delta DEF \sim \Delta XYZ$, then $\Delta ABC \sim \Delta XYZ$.

Example 7: Let $\Delta ABC \sim \Delta DEF$. As shown in Figure 4.21 below. Find

a. $m(\angle F)$ b. $m(\angle E)$ c. the length of BC.

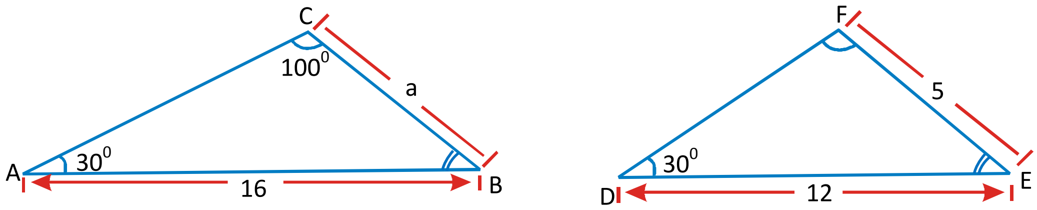


Figure 4.21

Solution: Given $\Delta ABC \sim \Delta DEF$ …. By definition 4.4

a. $\angle A \equiv \angle D$ if and only if $m(\angle A) = m(\angle D) = 30°$

$\angle B \equiv \angle E$ .... Marked angels

Therefore, $\angle C \equiv \angle F$ if and only if $m(\angle C) = m(\angle F) = 100°$

Therefore, $m(\angle F) = 100°$

b. $m(\angle A) + m(\angle B) + m(\angle C) = 180°$ ...Angle sum theorem

$30° + m(\angle B) + 100^0 = 180°$ .... Substitution

$m(\angle B) = 180° - 130° = 50°$

Therefore, $m(\angle E) = 50°$.

c. Given $\Delta ABC \sim \Delta DEF$

implies $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ .... Definition 4.4

$\frac{16}{12} = \frac{a}{5}$ .... Using the 1st and 2nd proportions

$12a = 80$ .... Cross multiplication

$a = \frac{80}{12}$ ....Dividing both sides by 12

Therefore, the length of BC$=\frac{80}{12}$ unit.

Example 8: In Figure 4.22 below, show that ΔABC and ΔLMN are similar.

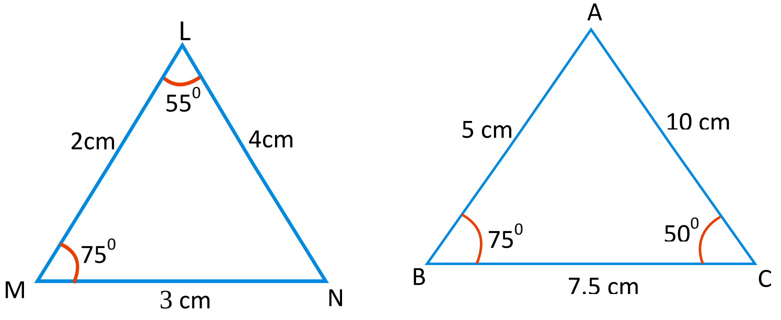


Figure 4.22

Solution: You begin by finding the unknown angels in the triangles. You know the size of two angles in each triangle and you also know that the sum of the angles of a triangle is 180^0. Therefore, it is easy to calculate the size of the unknown angles.

In ΔABC, $m(\angle ABC)+m(\angle BCA)+m(\angle CAB)=180°$.... Why?

$\Rightarrow 75^0+50^0+m(\angle CAB)=180^0$ .... Substitution

$\Rightarrow m(\angle CAB)=180^0-125^0$

$\Rightarrow m(\angle CAB)=55^0$

In ΔLMN, $m(\angle LMN)+m(\angle MNL)+m(\angle NLM)=180°$....Why?

$\Rightarrow 75^0+m(\angle MNL)+55^0=180^0$ .... Substitution

$\Rightarrow m(\angle MNL)=180^0-130^0$

$\Rightarrow m(\angle MNL)=50^0$

The corresponding angles are equal, to show the corresponding sides are in the same ratio. Let us check whether the corresponding sides are proportional or not. In short let us check.

$\Delta ABC \sim \Delta LMN$

$$\Rightarrow \frac{AB}{LM}=\frac{BC}{MN}=\frac{AC}{LN}=K \text{ (constant of proportionality)}$$

$$\frac{5cm}{2cm}=\frac{7.5cm}{3cm}=\frac{10cm}{4cm}=2.5$$

The corresponding sides are proportional with constant of proportionality equals 2.5. Therefore $\Delta ABC \sim \Delta LMN$….By definition 4.4.

Example 9: In Figure 4.23, $\Delta ABC \sim \Delta XYC$. If CX=6, CY=5, AX= 3 and AB= 8, find XY.

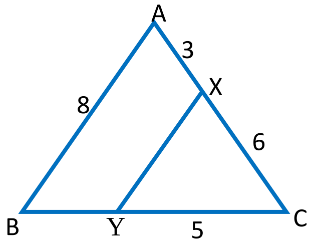


Figure 4.23

Solution:

By definition of similar triangles, we have:

$$\frac{AB}{XY}=\frac{BC}{YC}=\frac{AC}{XC}=K \text{ (constant proportionality)}$$

Thus $\frac{AB}{XY}=\frac{AC}{XC}$ .... Using the 1st and 3rd proportions

$\Rightarrow \frac{8}{XY}=\frac{AX+XC}{XC}$ since AX+XC=3+6=9

$\Rightarrow \frac{8}{XY}=\frac{9}{6}$

$\Rightarrow 9XY = 48$ .... Cross multiplication

$\Rightarrow XY = \frac{48}{9}$ ....Dividing both sides by 9.

Example 10: If $\Delta ABC \equiv \Delta DEF$, then

a. Is $\Delta ABC \sim \Delta DEF$?

b. Justify your answer.

Solution:

a. Yes

b. Suppose $\Delta ABC \equiv \Delta DEF$ are as shown in Figure 4.24 below:

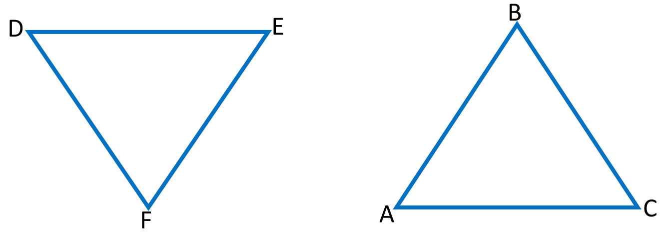


Figure 4.24

Then i. $\angle ABC \equiv \angle DEF$

$\angle BCA \equiv \angle EFD$ and

$\angle CAB \equiv \angle FDE$

Hence corresponding angles are congruent.

ii. $\overline{AB} \equiv \overline{DE}$, $\overline{BC} \equiv \overline{EF}$ and $\overline{AC} \equiv \overline{DF}$ implies

AB=DE, BC=EF, and AC=DF. Thus

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 1$$

Hence the corresponding sides are proportional. Since (from (i) and (ii))the corresponding sides are proportional with constant of proportionality 1, and also the corresponding angles are congruent, then the triangles are similar by definition 4.4. From example 10 above you can make the following generalization.

Note: Congruence is a similarity where the constant of proportionality is 1. This general fact is equivalent of the statement given below. If two triangles are congruent, then they are similar.

Exercise 4C

1. If $\Delta ABC \sim \Delta B'A'C'$, what are the pairs of corresponding angles and the pairs of corresponding sides?
2. If $\Delta ABC \sim \Delta A'B'C'$ and AC=20cm, A'C'=15cm, B'C'=12cm and A'B'=9cm, find the lengths of the other sides of ΔABC.

3. The sides of a triangle are 4cm, 6cm, and a cm respectively. The corresponding sides of a triangle similar to the first triangle are b cm, 12 cm and 8 cm respectively. What are the lengths a and b?
4. Are two similar triangles necessarily congruent? Why?
5. What is the length of the image of a 20cm long segment after central stretching with a scale factor $\frac{1}{2}$?
6. If $\Delta DEF \sim \Delta KLM$ such that DE =(2x + 2)cm, DF = (5x – 7)cm, KL = 2cm, KM = 3cm and EF= 10cm, then find LM.
7. In Figure 4.25 if $\Delta XYZ \sim \Delta WYP$, express d in terms of a, b and c.

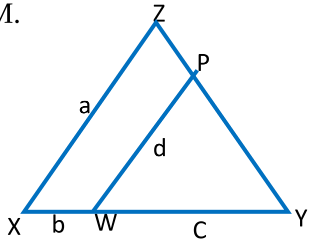


Figure 4.25

8. In Figure 4.26 below, if$\Delta ABC \sim \Delta XBZ$ with XB=6cm, BZ=5cm, CX=8cm and AC=7cm.
   What is the length of
   a. $\overline{BC}$?   b. $\overline{XZ}$?

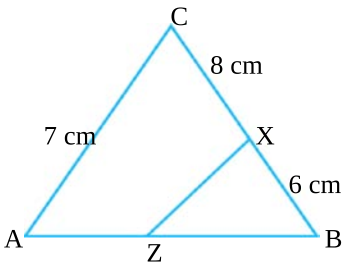


Figure. 4.26

Challenge Problems

9. Write down a pair of similar triangles in Figure 4.27 to the right. Find CD and AC, if AE is parallel to BD.

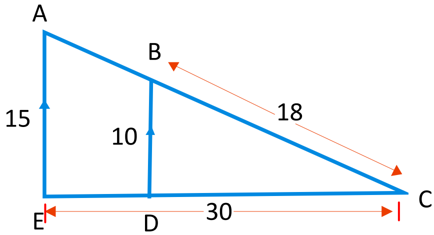


Figure. 4.27

4.2.2 Tests for Similarity of Triangles (SSS, SAS and AA)

Activity 4.3

Discuss with your teacher before starting the lesson.

1. Can you apply AA, SAS and SSS similarity theorems to decide whether a given triangles are similar or not?
2. Which of the following is (are) always correct?
   a. Congruent by SAS means similar by SAS.
   b. Similar by SAS means congruent by SAS.
   c. Congruent by SSS means similar by SSS.
   d. Similar by SSS means congruent by SSS.

? "But to decide whether two triangles are similar or not, it is necessary to know all the six facts stated in the definition 4.4?"

To prove similarity of triangles, using the definition of similarity means checking all the six conditions required by the definition. This is long and tiresome. Hence we want to have the minimum requirements which will guarantee us that the triangle are similar, i.e all the six conditions are satisfied. These short cut techniques are given as similarity theorems.

In this section you will see similarity theorems as you did see in grade 6 mathematics lessons congruence theorems for congruency of triangles.

Theorem 4.1: (AA-Similarity theorem)
If two angles of one triangle are congruent to the corresponding two angles of another triangle, then the two triangles are similar.

Example 11: In Figure 4.28 below $\angle R \cong \angle V$ and show that $\Delta RSW \sim \Delta VSB$.

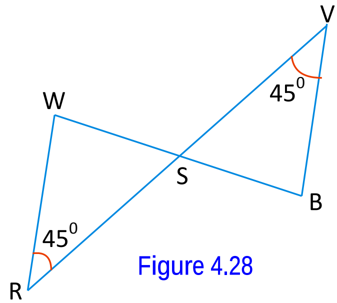


Figure 4.28

Proof:

Statements	Reasons
1. ∠R≅∠V	1. Degree measures are equal
2. ∠RSW≅∠VSB	2. Vertical opposite angles
3. ΔRSW~ΔVSB	3. AA similarity theorem

Example 12: In ΔABC and ΔXYZ if ∠ABC ≅ ∠XYZ, ∠ACB≅ ∠XZY, AB=8cm, AC=10cm and XY=4cm as shown in Figure 4.29 below; find the length of $\overline{XZ}$.

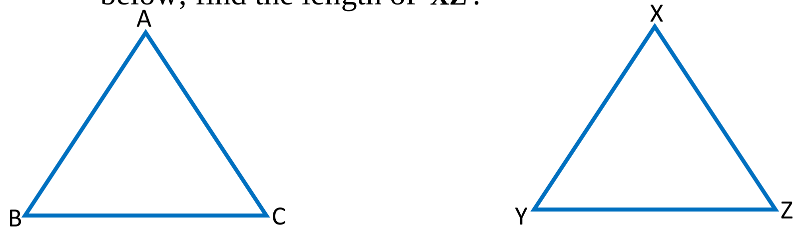


Figure 4.29

Solution: since ∠B ≅ ∠Y and ∠C ≅ ∠Z, we can say that

ΔABC ~ ΔXYZ by AA similarity theorem. Then,

$\frac{AB}{XY}=\frac{AC}{XZ}$ .... Definition of similar triangle.

Thus: $\frac{8}{4}=\frac{10}{XZ}$ ....Substitution

8XZ= 4×10 ....Cross multiplication

XZ= $\frac{40}{8}=5\text{cm}$

Theorem 4.2: (SAS-Similarity theorem)
If two sides of one triangle are proportional to the corresponding two sides of another triangle and their included angles are also congruent, then the two triangles are similar.

Example 13: In Figure 4.30 below, find DE.

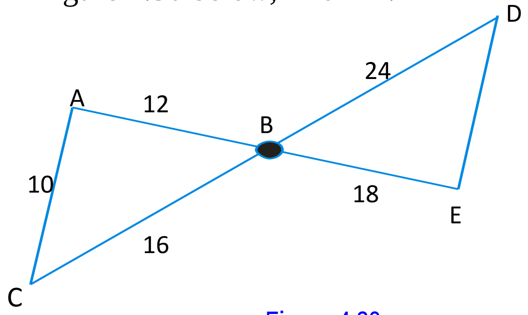


Figure 4.30

Solution:

1. $\angle ABC \cong \angle EBD$	1. Vertical opposite angles
2. $\frac{AB}{EB} = \frac{12}{18} = \frac{2}{3}$ and $\frac{BC}{BD} = \frac{16}{24} = \frac{2}{3}$	2. The ratio of the lengths of the corresponding sides are equal
3. $\Delta ABC \sim \Delta EBD$	3. SAS similarity theorem
4. $\frac{CA}{DE} = \frac{2}{3}$	4. Corresponding sides of similar triangles are proportional.
5. $\frac{10}{DE} = \frac{2}{3}$	5. Substitution
6. 2DE=30	6. Cross-product property
7. DE = 15 cm	7. solve for DE

Example 14: In Figure 4.31 below ΔABC and ΔDEF, are given where, $m(\angle A) = m(\angle D) = 55^0$, AB = 30 cm, AC = 100cm, DE = 15cm and DF = 50cm.

a. Are the two triangles similar?

b. Justify your answer.

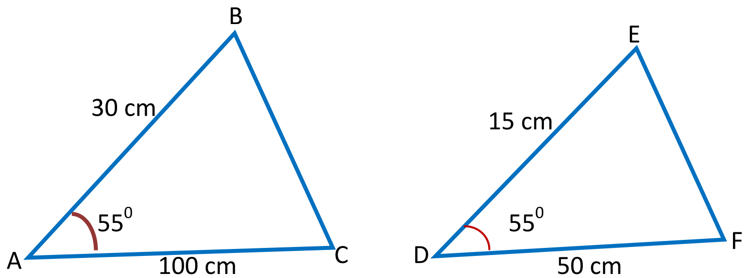


Figure 4.31

Solution:

a. Yes

b. Suppose ΔABC and ΔDEF are as shown in Figure 4.31 then,

$$\frac{AB}{DE}=\frac{30\text{cm}}{15\text{cm}}=2$$
$$\frac{AC}{DF}=\frac{100\text{cm}}{50\text{cm}}=2$$
$$\frac{AB}{DE}=\frac{AC}{DF}=2$$

Hence two sides of ΔABC are proportional to two corresponding sides of ΔDEF. Furthermore, $m(\angle A) = m(\angle D) = 55^0$, which shows that $m(\angle A) = m(\angle D)$. Thus the included angles between the proportional sides of ΔABC and ΔDEF are congruent. Therefore ΔABC~ΔDEF by SAS similarity theorem.

Theorem 4.3: (SSS-Similarity theorem)
If the three sides of one triangle are in proportion to the three sides of another triangle, then the two triangles are similar.

Example 15: Based on the given Figure 4.32 below decide whether the two triangles are similar or not. Write the correspondence.

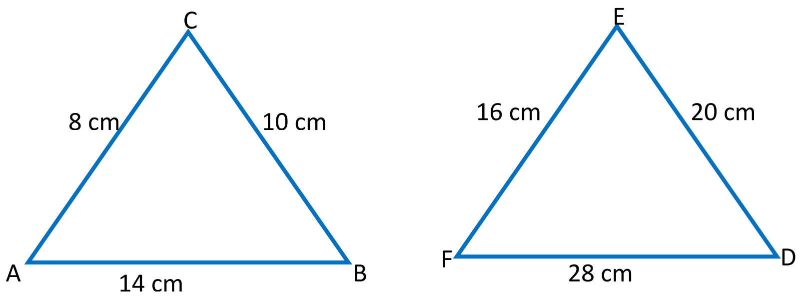


Figure 4.32

Solution:

$$\left.\begin{array}{l}\frac{AC}{FE}=\frac{8cm}{16cm}=\frac{1}{2}\\ \frac{CB}{ED}=\frac{10cm}{20cm}=\frac{1}{2}\\ \frac{AB}{FD}=\frac{14cm}{28cm}=\frac{1}{2}\end{array}\right\}$$

While finding proportional sides don't forget to compare the smallest with the smallest and the largest with the largest sides.

Hence $\frac{AC}{FE}=\frac{CB}{ED}=\frac{BA}{DF}=\frac{1}{2}$ or the sides are proportional.

Therefore $\Delta ABC \sim \Delta FDE$ .... By SSS similarity theorem.

From this you can conclude that: $\angle A \equiv \angle F$, $\angle B \equiv \angle D$ and $\angle C \equiv \angle E$.

Exercise 4D

1. If $\Delta ABC \sim \Delta XYZ$ and AC=10cm, AB=8cm and XY=4cm, find the length of $\overline{XZ}$.
2. Prove that any two equilateral triangles are similar.
3. In Figure 4.33 below determine the length x of the unknown side of ΔABC, if $\Delta ABC \sim \Delta DEF$.

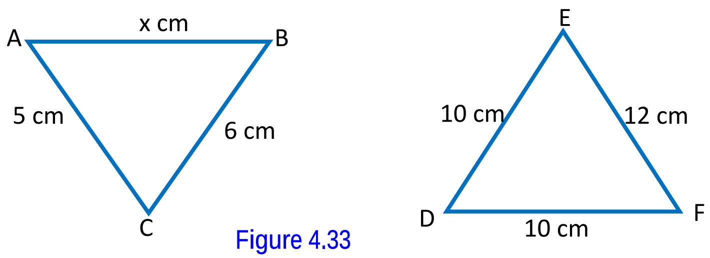


Figure 4.33

4. In Figure 4.34 below $\angle CAB \equiv \angle CDE$, AC=4cm, DC=5cm and DE=7cm. Determine the length of sides AB of ΔABC.

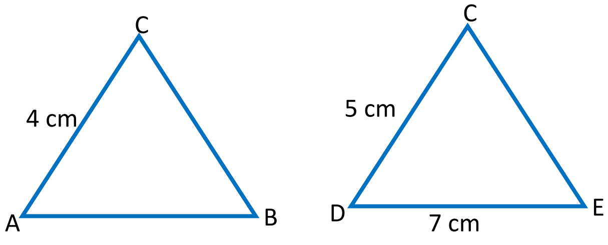


Figure 4.34

5. In Figure 4.35 of ΔABC, AC = 20cm, AB = 16cm, BC = 24cm.If D is a point on AC with CD = 15cm and, E is a point on BC with CE = 18cm, then:
   a. Show that ΔDEC~ΔABC.
   b. How long is DE?

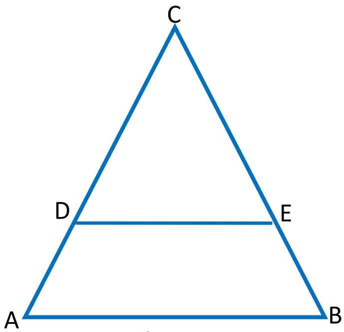


Figure 4.35

6. For any triangles ABC, if ∠A≡∠B, then show that ΔABC~ΔBAC.

7. Show that the corresponding altitudes of similar triangles ABC and PQR have the same ratio as two corresponding sides (See Figure 4.36).

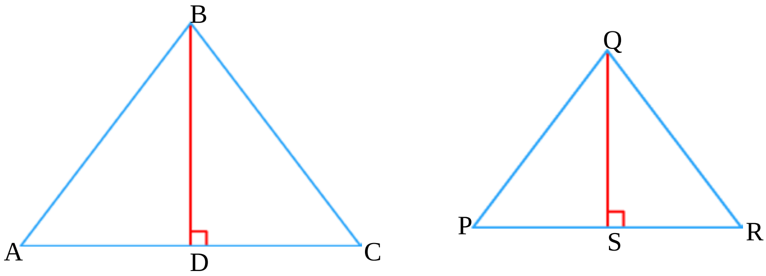


Figure 4.36

Challenge Problems

8. For the plane Figure 4.37 below $\overline{BE}$ and $\overline{AD}$ are altitude of ΔABC. prove that
   a. ΔADC~ΔBEC
   b. ΔAFE~ΔBFD

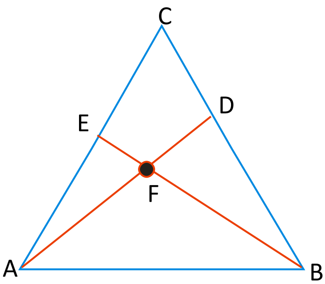


Figure 4.37

9. In Figure 4.38 to the right E is any point on $\overline{AB}$ and G is any point on $\overline{DC}$. If $\overline{AB} // \overline{DC}$, prove that:

   a. ΔDGF~ΔBEF. b. $\frac{DG}{EB} = \frac{DF}{FB}$.

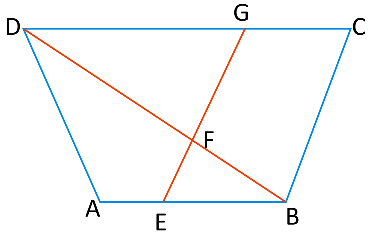


Figure. 4.38

10. In Figure 4.39 to the right determine the length AB of ΔABC.

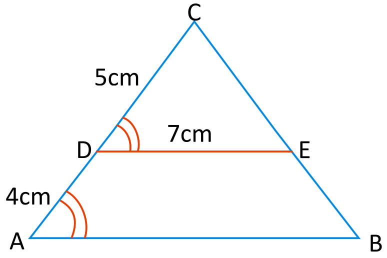


Figure 4.39

4.2.3 Perimeter and Area of Similar Triangles

Group work 4.3

1. Find the ratio of the areas of two similar triangles:
   a. if the ratio of their corresponding sides is $\frac{5}{4}$.
   b. if the ratio of their perimeters is $\frac{10}{9}$.
2. For the given Figure 4.40 below, find
   a. the area .
   b. the perimeter.

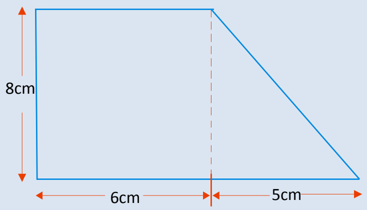


Figure 4.40

In lower grades you have seen how to find the perimeter and area of some special plane figures such as **triangles, rectangles, squares, parallelograms and trapeziums.** In the proceeding section of this unit you have been dealing with the areas and perimeters of similar plane figures. The perimeters and areas of similar plane figures have very interesting relations to their corresponding sides. You can compare the ratios of perimeters or that of the areas of similar polygons with out actually calculating the exact values of the perimeters or the areas. Look at the following example to help you clearly see these relations.

Example 16: In Figure 4.41 below if $\Delta ABC \sim \Delta XYZ$ with $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$.

Determine the relationship between:

a. The altitudes of the two triangles

b. The perimeters of the two triangles.

c. The areas of the triangles.

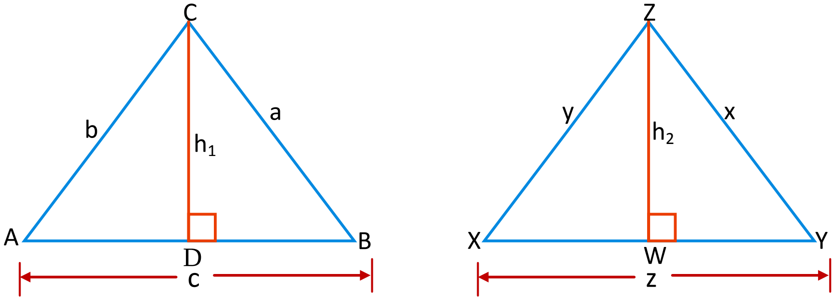


Figure. 4.41

Solution:

$\Delta ABC \sim \Delta XYZ$. let the constant of proportionality between their corresponding sides be K, i.e.

$\frac{a}{x} = k$ implies $a = kx$

$\frac{b}{y} = k$ implies $b=ky$

$\frac{c}{z} = k$ implies $c=kz$

$\frac{h_1}{h_2} = k$ implies $h_1=kh_2$

a. Let $\overline{CD}$ be the altitude of ΔABC from vertex C on $\overline{AB}$ and $\overline{ZW}$ be the altitude of ΔXYZ from vertex Z on $\overline{XY}$

Then $\angle CDB \cong \angle ZWY$ …. Both are right angles.

$\angle B \cong \angle Y$ …… Corresponding angles or similar triangles.

Therefore, $\Delta CDB \sim \Delta ZWY$ … By AA similarity theorem.

Thus $\frac{CD}{ZW} = \frac{CB}{ZY}$ ……. Definition of similar triangles.

$\Rightarrow \frac{h_1}{h_2} = \frac{a}{x}$ ……. Substitution

$\Rightarrow \frac{h_1}{h_2} = k$ …….. Since $\frac{a}{x} = k$ proportional sides

$\frac{h_1}{h_2} = k$ implies $h_1 = kh_2$

b. $P(\Delta ABC)=a+b+c$

$=kx+ky+kz$

$=k(x+y+z)$

and $p(\Delta xyz) = x+y+z$

then $\frac{P(\Delta ABC)}{P(\Delta XYZ)} = \frac{K(x+y+z)}{x+y+z} = k$

Hence the ratio of the perimeters of the two similar triangles is "k" which is equal to the ratio of the lengths of any pair of corresponding sides.

c. $a(\Delta ABC) = \frac{1}{2}\ c.h_1$

$= \frac{1}{2}(kzh_1)$

$= \frac{1}{2}(kz.kh_2)$

and $a(\Delta xyz) = \frac{1}{2}zh_2$

then $\frac{a(\Delta ABC)}{a(\Delta XYZ)} = \frac{\frac{1}{2}kz.kh_2}{\frac{1}{2}zh_2} = k^2$

Hence the **ratio of the areas of the two similar triangles** is k^2, the square of the ratio of the lengths of any pair of corresponding sides. The above examples will lead us to the following two important generalization which could be stated as theorems.

Theorem 4.4: If the ratios of the corresponding sides of two similar polygons is k, then the ratio of their perimeters is given by $\frac{P_1}{P_2} = \frac{S_1}{S_2} = k$.

Theorem 4.5: If the ratios of the corresponding sides of two similar polygons is $\frac{S_1}{S_2} = k$, then the ratio of their areas, is given by: $\frac{A_1}{A_2} = \left(\frac{S_1}{S_2}\right)^2 = k^2$.

Example 17: Find the ratio of the areas of two similar triangles,

a. If the ratio of the corresponding sides is $\frac{5}{4}$.

b. If the ratio of their perimeters is $\frac{10}{9}$.

Solution:

Let A_1, A_2 be areas of two similar triangles, P_1, P_2 be the perimeters of the two triangles and S_1, S_2 be their corresponding sides.

a. $\frac{A_1}{A_2} = \left(\frac{S_1}{S_2}\right)^2$ ....Theorem 4.5

$\frac{A_1}{A_2} = \left(\frac{5}{4}\right)^2$ .... Substitution

Therefore, $\frac{A_1}{A_2} = \frac{25}{16}$.

b. $\frac{A_1}{A_2} = \left(\frac{P_1}{P_2}\right)^2$ .... Theorem 4.4 and 4.5

$\frac{A_1}{A_2} = \left(\frac{10}{9}\right)^2$ .... Substitution

Therefore, $\frac{A_1}{A_2} = \frac{100}{81}$.

Example 18: The areas of two similar polygons are $80cm^2$ and $5cm^2$. If a side of the smaller polygon is 2cm, find the corresponding sides of the larger polygons.

Solution:

Let A_1 and A_2 be areas of the two polygons and S_1, S_2 be their corresponding sides, then

$\left(\frac{S_1}{S_1}\right)^2 = \frac{A_1}{A_2}$ .... Theorem 4.5

$\left(\frac{S_1}{S_1}\right)^2 = \frac{80}{5}$ .... Substitution

$$\frac{S_1^2}{2^2} = \frac{80}{5}$$

$$\frac{S_1^2}{4} = 16$$

$$S_1^2 = 64$$

$$S_1 \times S_1 = 8 \times 8$$

$$S_1 = 8cm$$

Therefore, the corresponding sides of the larger polygon is 8cm.

Example 19: The sum of the perimeters of two similar polygon is 18cm. The ratios of the corresponding sides is 4:5. Find the perimeter of each polygon.

Solution:

Let S_1 and S_2 be the lengths of the corresponding sides of the polygon and P_1 and P_2 be their perimeters.

$P_1+P_2=18$

$P_1=18-P_2$ .... Solve for P_1

Thus $\frac{P_1}{P_2} = \frac{S_1}{S_2}$ ....Theorem 4.4

$\frac{18-P_2}{P_2} = \frac{4}{5}$ .... Substitution

$4P_2=90-5P_2$ .... Cross multiplication

$9P_2=90$

$P_2=10$ .... Divides both sides by 9.

Therefore, when $P_2=10$

$$P_1+P_2=18$$
$$P_1+10=18$$
$$P_1=8$$

Exercise 4E

1. In two similar triangles, find the ratio of:
   a. corresponding sides, if the areas are $50cm^2$ and $98cm^2$.
   b. the perimeter, if the areas are $50cm^2$ and $16cm^2$.
2. Two triangles are similar. The length of a side of one of the triangles is 6 times that of the corresponding sides of the other. Find the ratios of the perimeters and the area of the triangles.
3. The sides of a polygon have lengths 5, 7, 8, 11 and 19 cm. The perimeter of a similar polygon is 75cm. Find the lengths of the sides of larger polygon.
4. A side of a regular six – sided polygon is 8cm long. The perimeter of a similar polygon is 60cm. What is the length of a side of the larger polygon?
5. The ratio of the sides of two similar polygon is 3:2. The area of the smaller polygon is $24cm^2$. What is the area of the larger polygon?
6. Two trapeziums are similar. The area of one of the trapeziums is 4 times that of the other. Determine the ratios of the perimeters and the corresponding side lengths of the trapeziums.
7. Two triangles are similar. The length of a side of one of the triangles is 4 times that of the corresponding side of the other. Determine the ratios of the perimeters and the areas of the polygon.

Challenge Problems

8. Two pentagons are similar. The area of one of the pentagons is 9 times that of the other. Determine the ratios of the lengths of the corresponding sides and the perimeters of the pentagons.

9. Two triangles are similar. The length of a side of one of the triangles 2 times that of corresponding sides of the other. The area of the smaller triangle is 25sq.cm. Find the area of the larger triangle.
10. The lengths of the sides of a quadrilateral are 5cm, 6cm, 8cm and 11cm. The perimeter of a similar quadrilateral is 20cm. Find the lengths of the sides of the second quadrilateral.
11. The picture represents a man made pool surrounded by a park. The two quadrilateral are similar and the area of the pool is 1600 sq.cm. What is the area of the park of A'B' is four times the length of AB?

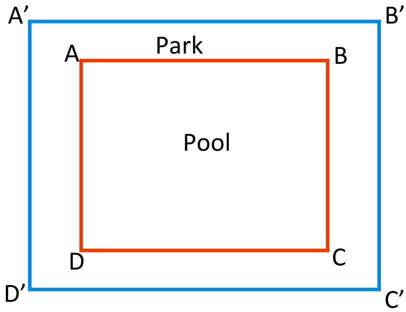


Figure 4.42

Summary For Unit 4

1. *Similar geometric figures are figures which have the same shape.*
2. *Two polygons are similar if:*
   i. *their corresponding sides are proportional.*
   ii. *their corresponding angles are congruent.*
3. *Under an enlargement*
   a. *Lines and their images are parallel.*
   b. *Angles remain the same.*
   c. *All lengths are increased or decreased in the same ratio. .*
4. ***Scale factor-*** *the ratio of corresponding sides usually expressed numerically so that:*

$$\text{scale factor} = \frac{\text{length of line on the enlargement}}{\text{length of line on the original}}$$

5. *ΔABC similar to ΔDEF if:*
   i. *their corresponding sides are proportional.*
   ii. *their corresponding angles are congruent.*
6. ***AA Similarity theorems***

   If two angles of one triangle are congruent to the corresponding two angles of another triangle, then the two triangles are similar.

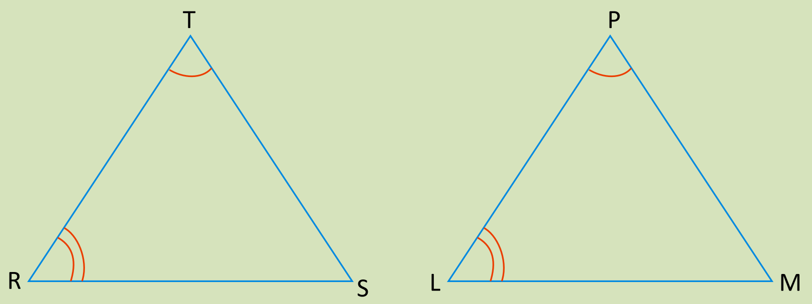


Figure 4.43

In the above Figure 4.43 you have:

- $\angle T \equiv \angle P$ *if and only if* $m(\angle T)=m(\angle P)$
- $\angle R \equiv \angle L$ *if and only if* $m(\angle R=m(\angle L)$
- $\Delta TRS \sim \Delta PLM$ .... *by AA similarity.*

7. ***SAS Similarity theorem***

If two sides of one triangle are proportional to the corresponding two sides of another triangle and their included angles are also congruent, then the two triangles are similar.

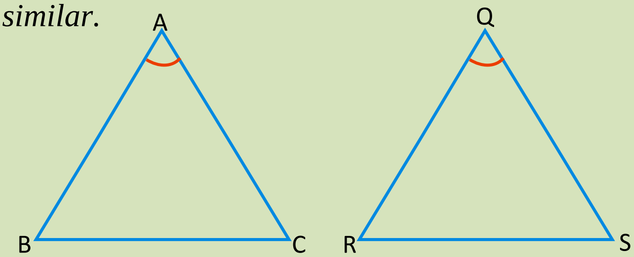


Figure 4.44

- $\angle A \equiv \angle Q$ *if and only if* $m(\angle A)=m(\angle Q)$
- $\frac{AB}{QR}=\frac{AC}{QS}$

Therefore $\Delta ABC \sim \Delta QRS$ .... *by SAS similarity.*

8. ***SSS Similarity theorem***

If the three sides of one triangle are in proportion to the three sides of another triangle, then the two triangles are similar.

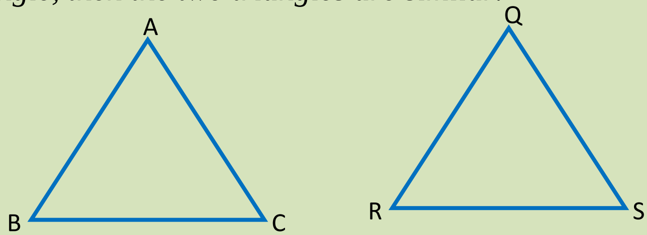


Figure 4.45

In the above Figure 4.45 you have:

$$\frac{AB}{QR}=\frac{BC}{RS}=\frac{AC}{QS}$$

Therefore, $\Delta ABC \sim \Delta QRS$ .... *SSS Similarity.*

9. In Figure 4.46, If $\Delta ABC \sim \Delta DEF$ with constant of proportionality k, then

a. $P(\Delta ABC)=KP(\Delta DEF)$

b. $a(\Delta ABC)=K^2 \times a(\Delta DEF)$

c. $AB = KDE$,

$BC = KEF$

$AC = KDE$

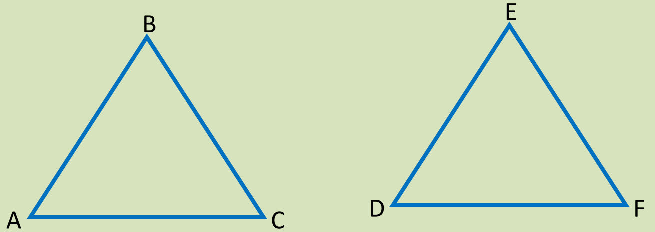


Figure 4.46

Miscellaneous Exercise 4

I Write true for the correct statements and false for the incorrect one.

1. $\overline{AB} \equiv \overline{CD}$ if and only if AB=CD.
2. ∠ABC≅∠DEF if and only if m(∠ABC)=m(∠DEF).
3. All rhombuses are similar.
4. All congruent polygons are similar.
5. All Isosceles triangles are similar.
6. Any two equilateral triangles are similar.

II Choose the correct answer from the given alternatives.

7. In Figure 4.47 given below $\overline{PQ} \perp \overline{QT}, \overline{ST} \perp \overline{QT}$ and P, R and S are on the same line. If ΔPQR~ΔSTR, then which of the following similarity theorems supports your answer?
   a. SAS Theorem
   b. AA Theorem
   c. SSS Theorem
   d. None

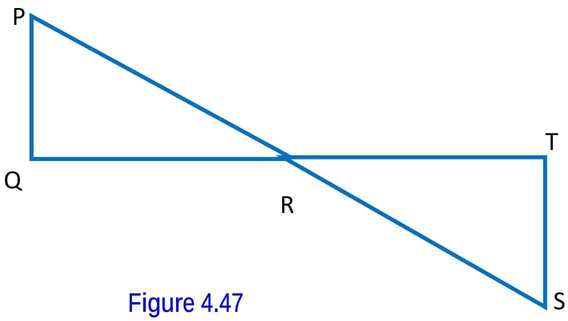


Figure 4.47

8. Given ΔABC and ΔDEF, if $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then which of the following postulates or theorem shows that ΔABC is similar to ΔDEF?
   a. AA similarity theorem.
   b. AAA similarity theorem.
   c. SAS similarity postulate.
   d. SSS similarity theorem.

9. Which of the following plane figures are not necessarily similar to each other?
   a. two equilateral triangles.
   b. two isosceles triangles.
   c. two circles.
   d. two squares.

10. Which of the following is different in meaning from $\Delta ADF \sim \Delta LMN$?
   a. $\Delta DFA \sim \Delta NML$
   b. $\Delta FAD \sim \Delta MNL$
   c. $\Delta AFD \sim \Delta LMN$
   d. $\Delta DAF \sim \Delta NLM$

11. In Figure 4.48 below MN//YZ. If XN = 10cm, NZ = 5cm and MY = 4cm, then what is the length of $\overline{XY}$?
   a. 9cm
   b. 15 cm
   c. 12 cm
   d. 18 cm

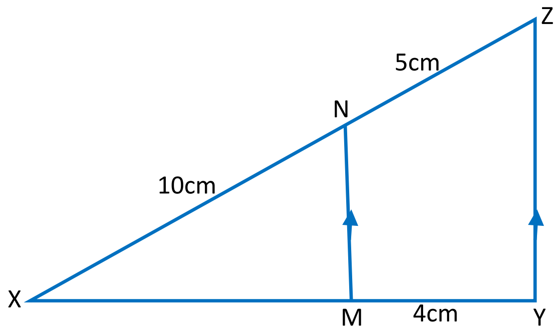


Figure 4.48

III Work out problems

12. Rectangle ABCD is similar to rectangle PQRS. Given that AB=14cm, BC=8cm and PQ=21 cm, calculate the length of QR.

13. A football field measures 100m by 72m. A school marks a football field similar in shape to a full size foot ball field but only 30m long. What is its width?

14. In Figure 4.49 below $\angle QRP \cong \angle XYZ$ and show that $\Delta QRP \sim \Delta XYZ$.

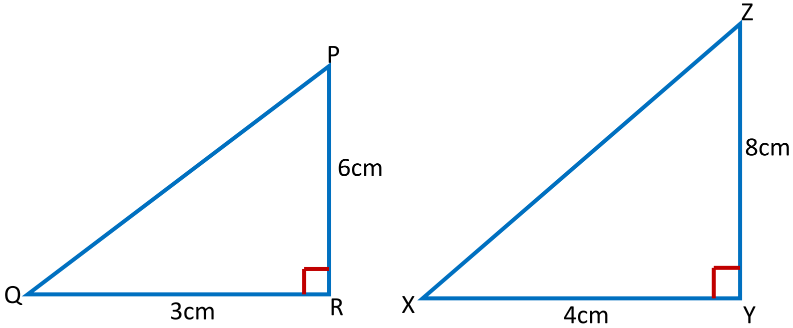


Figure 4.49

15. The sides of a polygon are 3cm, 5cm, 6cm, 8cm and 10cm. The perimeter of a similar polygons is 48cm. Find the sides of the second polygon.

16. In Figure 4.50 to the right $\overline{DC} // \overline{AB}$,

prove that $\frac{AO}{OD} = \frac{BO}{OC}$

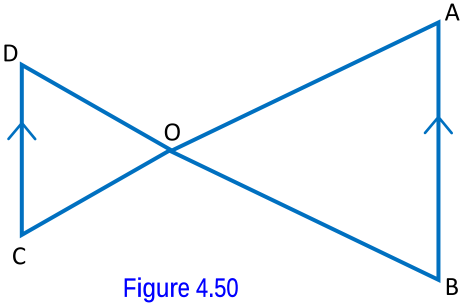


Figure 4.50

17. A piece of wood is cut as shown in Figure 4.51 below. The external and internal edge of the wood are similar quadrilaterals:

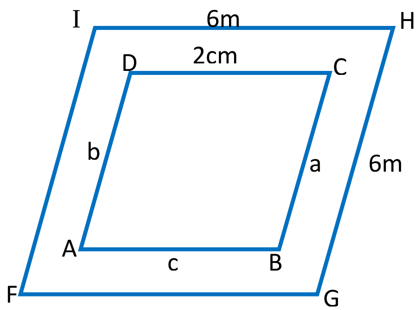


Figure 4.51

i.e. ABCD~ FGHI. The lengths of the sides are indicated on the figure. How long are the internal edges marked as a, b, and C?

UNIT 5 CIRCLES

Unit outcomes

After Completing this unit, you should be able to:

- have a better understanding of circles.
- realize the relationship between lines and circles.
- apply basic facts about central and inscribed angles and angles formed by intersecting chords to compute their measures.

Introduction

In the previous grades you had learnt about circle and its parts like its center, radius and diameter. Now in this unit you will learn about the positional relationship of a circle and lines followed by chords and angles formed inside a circle and how to compute their degree measures of such angles.

5.1 Further On Circles

Activity 5.1

Discussed with your teacher orally.

Define and show by drawing the following key terms:

a. circles c. diameter e. circumference of a circle

b. radius d. chord

Now in this lesson you will discuss more about parts of a circle i.e **minor arc** and **major arc**, **sector** and **segment of a circle**, **tangent** and **secant of a circle** and **center of a circle** by construction.

Parts of a circle

Group Work 5.1

1. a. Draw a circle of radius 4cm.
   b. Draw a diameter in your circle. The diameter divides the circle in to two semicircles.
   c. Colour the two semicircles indifferent colours.
   d. Draw a minor arc in your circle and label your minor arc.
   e. Draw a major arc in your circle and label your major arc.
2. A circle has a diameter of 6cm.
   a. write down the length of the radius of the circle.
   b. Draw the circle.
   c. Draw a chord in the circle.

Definition 5.1: The set of points on a circle (part of a circle) contained in one of the two half-planes determined by the line through any two distinct points of a circle is called **an arc of a circle.**

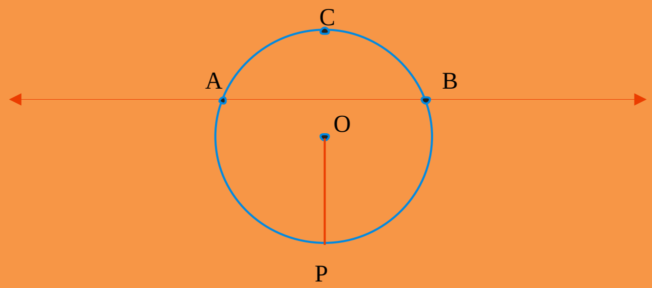


Figure 5.1 Circle

The center of the circle is O and PO is the radius. The part of the circle determined by the line through points A and B is an arc of the circle. In Figure 5.1 above arc ACB is denoted by $\overset{\frown}{ACB}$ or arc APB is denoted by $\overset{\frown}{APB}$.

A. Classification of Arcs

i. Semi-circle: Is half of a circle whose end points are the end points of a diameter of the circle and measure is 180^0.

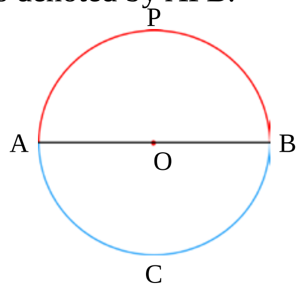


Figure 5.2 $\overset{\frown}{APB}$ and $\overset{\frown}{ACB}$ are a semi-circles.

ii. Minor arc: is the part of a circle which is less than a semi-circle.

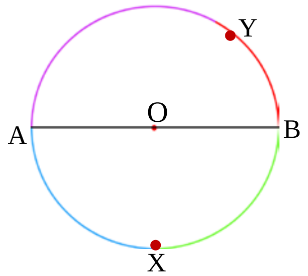


Figure 5.3 $\overset{\frown}{AX}$, $\overset{\frown}{BY}$, $\overset{\frown}{AY}$ and $\overset{\frown}{BX}$ are a minor arcs

iii. Major arc: is the part of a circle which is greater than a semi-circle.

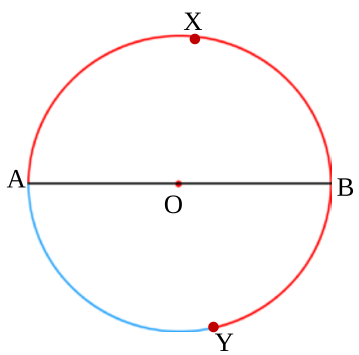


Fig. 5.4 $\overset{\frown}{AXBY}$ and $\overset{\frown}{BYAX}$ are major arcs.

Example 1: In Figure 5.5 below determine whether the arc is a minor arc, a major arc or a semicircle of a circle O with diameters $\overline{AD}$ and $\overline{BE}$.

a. $\overset{\frown}{AFB}$
b. $\overset{\frown}{ABD}$
c. $\overset{\frown}{BED}$
d. $\overset{\frown}{CAE}$
e. $\overset{\frown}{CDE}$
f. $\overset{\frown}{BCD}$
g. $\overset{\frown}{AED}$
h. $\overset{\frown}{ABC}$

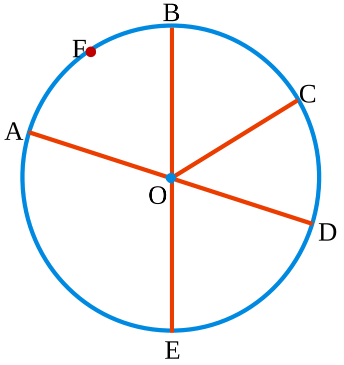


Figure 5.5 Circle

Solution:

a. minor arc
b. semi-circle
c. major arc
d. major arc
e. minor arc
f. minor arc
g. semi-circle
h. minor arc

B. Sector and segments of a circle

Definition 5.2: A sector of a circle is the region bounded by two radii and an intercepted arc of the circle.

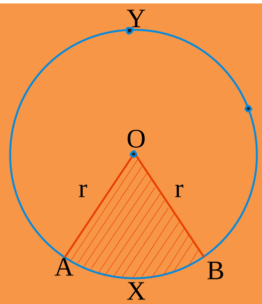


Figure 5.6 Circle

In Figure 5.6 above, the shaded region AOB and the unshaded region AYB are sectors of the circle.

Definition 5.3: *A segment* of a circle is the region bounded by a chord and the arc of the circle.

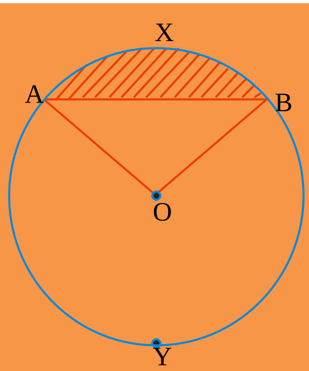


Figure 5.7 Circle

In Figure 5.7 above the shaded region AXB and the unshaded region AYB are segments of the circle.

C. Positional relations between a circle and a line

A circle and a line may be related in one of the following three ways.

1. The line may not intersect the circle at all.

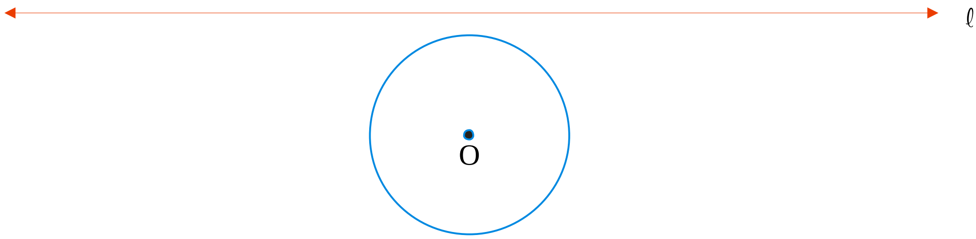


Figure 5.8 Circle

2. The line may intersect the circle at exactly one point.

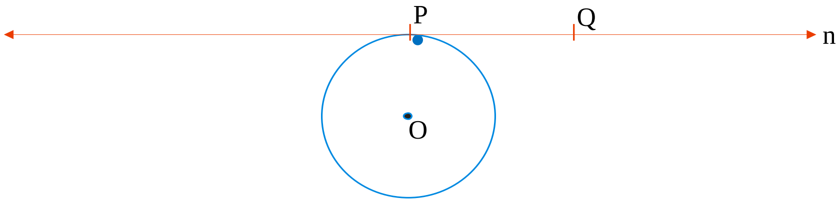


Figure 5.9 Circle

3. The line may intersect the circle at two points

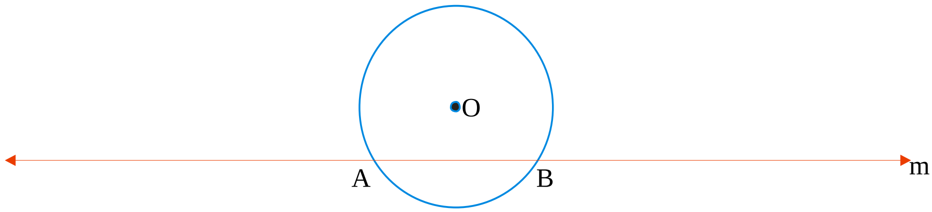


Figure 5.10 Circle

Definition 5.4: a. If a line intersect a circle at exactly one point, then the line is called a tangent of the circle.

b. The point at which it intersect the circles is called point of tangency.

c. If a line intersect a circle at two points then the line is called a secant of the circle.

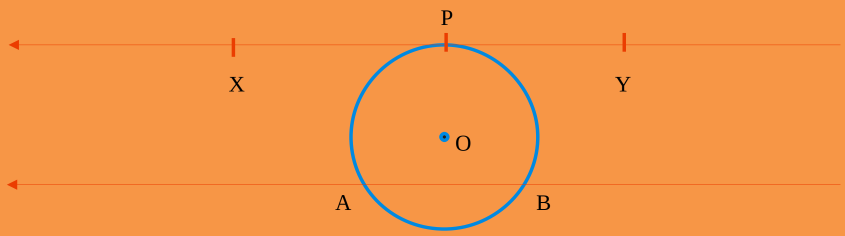


Figure 5.11 Circle

In Figure 5.11 above P is the point of **tangency**, $\overleftrightarrow{XY}$ is a tangent to the circle O and $\overleftrightarrow{AB}$ is a **secant** line to the circle O.

D. Construction

To find the center of a circle by construction the following steps is important:

Step i : Draw a circle by using coins

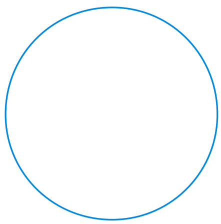


Figure 5.12 circle

Step ii : Draw a chord $\overline{AB}$

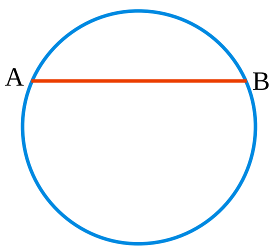
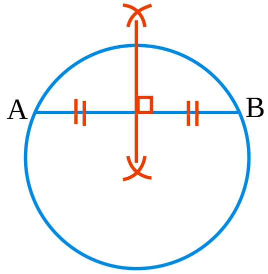


Figure 5.13 circle

Step iii: Construct the perpendicular bisector of $\overline{AB}$



Figurer 5.14 circle

Step iv: Draw another chord $\overline{CD}$.

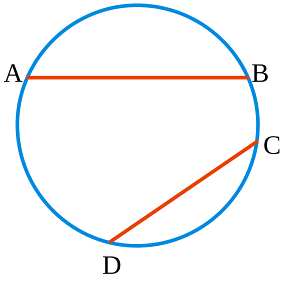


Figure 5.15 circle

Step v: Construct the perpendicular bisector of $\overline{CD}$.

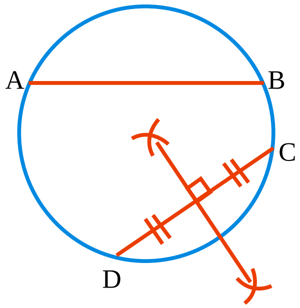


Figure 5.16 circle

Step vi: The perpendicular bisectors of $\overline{AB}$ and $\overline{CD}$ intersect at O, the centre of the circle.

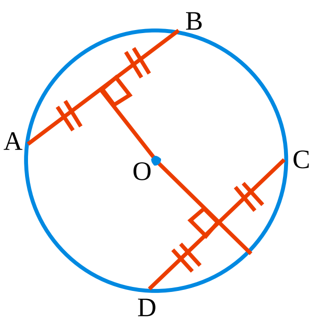


Figure 5.17 The required circle

Exercise 5A

1. Write true for the correct statements and false for the incorrect ones of each of the following.
   a. A secant of a circle contains chord of the circle.
   b. A secant of a circle always contains diameter of the circle.
   c. A tangent to a circle contains an interior point of the circle.
   d. A tangent to a circle can pass through the center of the circle.

2. In Figure 5.18 below A is an interior point of circle O. B is on the circle and C is an exterior point. Write correct for the true statements and false for the incorrect ones of each of the following.
   a. You can draw a secant line through point C.
   b. You can draw a secant line through point B.
   c. You can draw a tangent line through point A.
   d. You can draw tangent line through point C.

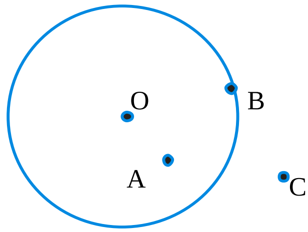


Figure 5.18 Circle

3. Consider the following Figure 5.19 to complete the blank space.
   a. _______ is tangent to circle O.
   b. _______ is secant to circle O.
   c. _______ is tangent to circle Q.
   d._______ is secant to circle Q.
   e. _______ is the common chord to circle O and Q.

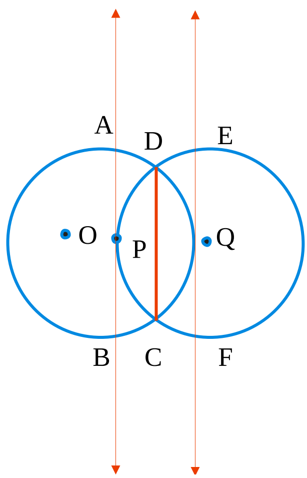


Figure 5.19 Circle

5.2 Angles in the Circle

Now in this lesson you will discuss more about central angle, inscribed angle, Angles formed by two intersecting chords and cyclic quadrilaterals.

5.2.1 Central Angle and Inscribed Angle

Group Work 5.2

1. **What is central angle?**
2. **What is inscribed angle?**
3. **Explain the relationship between the measure of the inscribed angle and measure of the arc subtends it.**
4. **In the given Figure 5.20 below m(∠CAO)=30° and m(∠CBO)=40°. Find m(∠ACB) and m(∠AOB).**

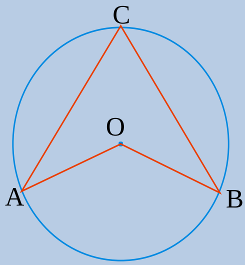


Figure 5.20

5. **If in Figure 5.21 arc BD is two times the arc AC, find ∠ BAD.**

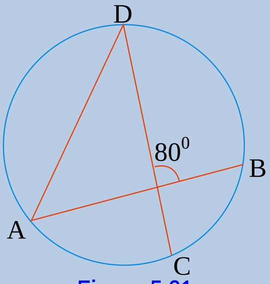


Figure 5.21

6. **O is the center of the circle. The straight line AOB is parallel to DC. Calculate the values of a, b and c.**

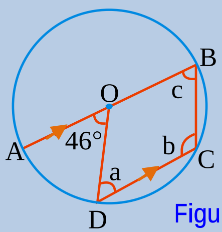


Figure 5.22

Definition 5.5: A Central angle of a circle is an angle whose vertex is the center of the circle and whose sides are radii of the circle.

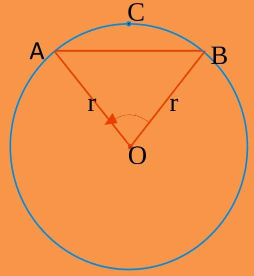


Figure 5.23 ∠AOB is a central

Note: 1. $\overset{\frown}{ACB}$ is said to be intercepted by ∠AOB and ∠AOB is said to be subtended by $\overset{\frown}{ACB}$.

2. The chord $\overline{AB}$ is said to subtend $\overset{\frown}{ACB}$ and $\overset{\frown}{ACB}$ is said to be subtended by chord $\overline{AB}$.

3. Chord $\overline{AB}$ subtends ∠AOB.

4. The measure of the central∠AOB is equal to the measure of the intercepted $\overset{\frown}{ACB}$. i.e m (∠AOB) = $\overset{\frown}{ACB}$.

Fact:- If the measure of the central angle is double or halved, the length of the intercepted arc is also doubled or halved. Thus you can say that the length of an arc is directly proportional to the measure of the central angle subtended by it. Hence you can use this fact to determine the degree measure of an arc by the central angle under consideration.

Definition 5.6: An inscribed angle is an angle whose vertex is on a circle and whose sides contain chords of the circle.

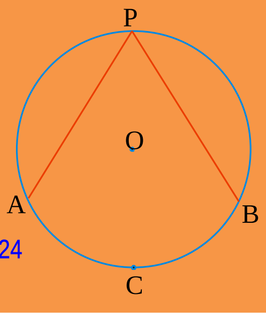


Figure 5.24

In Figure 5.24 above $\angle APB$ is inscribed angle of circle O. we say $\angle APB$ is inscribed in $\overset{\frown}{ACB}$ and $\overset{\frown}{ACB}$ subtends $\angle APB$.

Note: In Figure 5.25 below, the relationship between the measure of the central angle and inscribed angle by the same arc is given as follows:

1. The measure of the inscribed angle is half of the measure of central angle.
2. The measure of the inscribed angle is half of the measure of the arc subtends it.

$m(\angle ABC) = \frac{1}{2}\, m(\angle AOC)$

$m\,(\angle ABC) = \frac{1}{2}\, m(\overset{\frown}{ADC})$

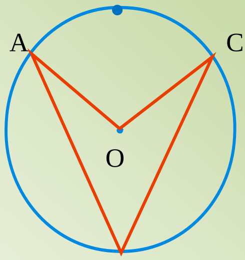


Figure 5.25

3. In Figure 5.26 to the right the relationship of inscribed angles subtended by the same arc is i.e $m\,(\angle ABE)=m(\angle ACE)=m(\angle ADE)$

$= \frac{1}{2}\, m\,(\overset{\frown}{AXE})$

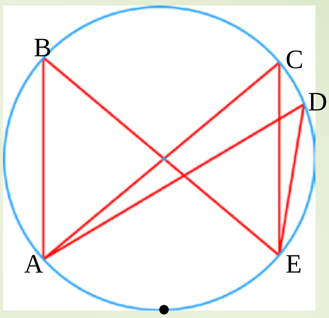


Figure. 5.26

Examples 2: In Figure 5.27 to the right, O is the center of the circle. If $m(\angle AOC)=52^0$, find $m\,(\angle ABC)$ and $m\,(\overset{\frown}{AC})$.

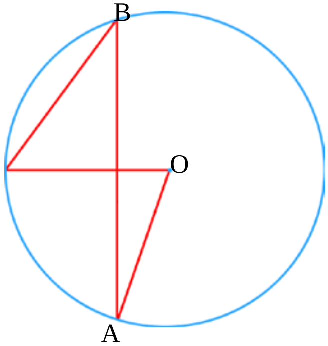


Figure 5.27

Solution:

$m\,(\angle AOC) = m\,(\overset{\frown}{AC}) = 52^0$ and

$m\,(\angle ABC) = \frac{1}{2} m\;(\overset{\frown}{AC})$

$= \frac{1}{2}(52^0)$

$= 26^0$

Examples 3: In Figure 5.28 to the right, O is the center of the circle, m (∠ABC) = 65^0, and m (∠AOE) =70^0, find m (∠CFE).

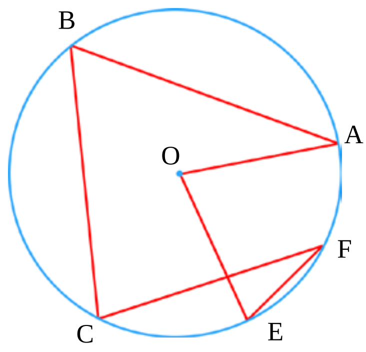


Figure 5.28

Solution:

m(∠AOE)=70^0 .... Given

m(∠ABC)=65^0 .... Given

$m(\angle ABC)= \frac{1}{2} m\,(\overset{\frown}{AFEC})$

$65^0 = \frac{1}{2} m\,(\overset{\frown}{AFEC})$

$m\,(\overset{\frown}{AFEC}) = 130^0$

$m\,(\angle AOE) = m\,(\overset{\frown}{AFE}) = 70°$

Thus $m\,(\overset{\frown}{EC}) = m(\overset{\frown}{AFEC}) - m\,(\overset{\frown}{AFE})$

$= 130^0 - 70^0$

$= 60^0$

Therefore, $m\,(\angle CFE) = \frac{1}{2} m\,(\overset{\frown}{EC})$

$= \frac{1}{2}(60^0)$

$= 30^0$

Examples 4: In Figure 5.29 to the right, O is the center of the circle,

m (∠AQB) = 35^0

Find a. m (∠AOB)

b. m (∠APB)

c. m (∠ARB)

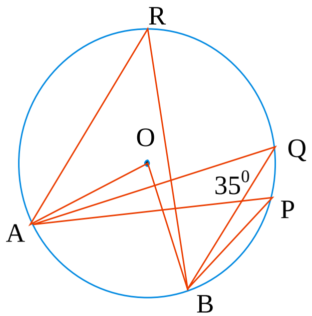


Figure 5.29

Solution:

m (∠AOB) =m $\overset{\frown}{AB}$

a. $m(\angle AQB)=\frac{1}{2}(m\angle AOB)$

$\Rightarrow m(\angle AOB)=2m(\angle AQB)$

$=2(35^0)$

$=70^0$

b. $m(\angle APB)=\frac{1}{2}m(\angle AOB)$

$=\frac{1}{2}(70^0)=35^0$

c. $m(\angle ARB)=\frac{1}{2}m(\angle AOB)$

$=\frac{1}{2}(70^0)=35^0$

Examples 5: In Figure 5.30 to the right, find the values of the variables.

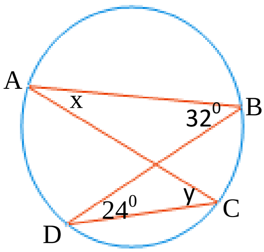


Figure 5.30

Solution:

$m(\angle ABD)=\frac{1}{2}m(\overset{\frown}{AD})$

$\Rightarrow m(\overset{\frown}{AD})=2m(\angle ABD)$

$=2\times 32^0$

$=64^0$

$m(\angle ACD)=\frac{1}{2}m(\overset{\frown}{AD})$

$y=\frac{1}{2}(64^0)$

$y=32^0$

$m(\angle BDC)=\frac{1}{2}m(\overset{\frown}{BC})$

$$\Rightarrow m(\overset{\frown}{BC}) = 2m(\angle BDC)$$
$$= 2 \times 24^0$$
$$= 48^0$$

$$m(\angle BAC) = \frac{1}{2} m(\overset{\frown}{BC})$$

$$x = \frac{1}{2}(48^0)$$

$$x = 24^0$$

5.2.2 Theorems on Angles in A Circle

You are already familiar with central angles and inscribed angles of a circle. Under this sub-section you will see some interesting result in connection with central and inscribed angles of a circle.

It is well know that the measure of a central angle is equal to the measure of the intercepted arc. But, a central angle is not the only kind of angle that can intercept an arc.

Theorem 5.1: The measure of an inscribed angle is equal to one half of the measure of its intercepted arc.

This important theorem proved in three cases. But here you can consider only the first case.

Proof: Given an inscribed angle ABC with sides $\overleftrightarrow{BC}$ passing through the center O.

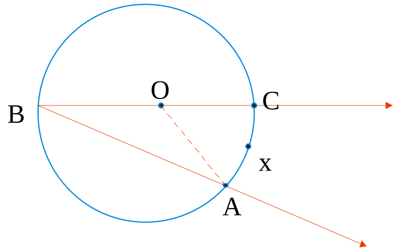


Figure 5.31

We want to show that: $m(\angle ABC) = \frac{1}{2} m(\overset{\frown}{AXC})$

Statements	Reasons
1. Draw $\overline{OA}$	1. Through two points there is exactly one line.
2. ΔBOA is isosceles	2. $\overline{OB} \equiv \overline{OA}$ (radii).
3. $\angle OBA \cong \angle OAB$	3. Base angles of isosceles triangle
4. $m(\angle OBA)+m(\angle OAB)=m(\angle AOC)$	4. $\angle AOC$ is supplementary to $\angle OBA$, $m(\angle OAB)$, + $m(\angle OBA) + m(\angle BOA) = 180^{o}$.
5. $m(\angle ABO)= \frac{1}{2} m(\angle AOC)$	5. since $\angle BAO \cong \angle ABO$.
6. $m(\overset{\frown}{AXC})=m(\angle AOC)$	6. central angle AOC intercepts $\overset{\frown}{AXC}$.
7. $m(\angle ABC)=m(\angle ABO)$	7. Naming the same angle.
8. $m(\angle ABC) = \frac{1}{2} m(\overset{\frown}{AXC})$	8. Substitution in step 5 .

Example 6: In Figure 5.32 below, O is the centre of a circle.

$m(\angle QPT)=54^0$ and $m(\angle TSQ)=21^0$.

Find: $m(\angle ROS)$.

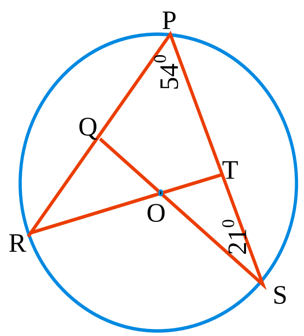


Figure 5.32

Solution:

$$m(\angle RPS) = \frac{1}{2} m(\overset{\frown}{RS})$$

$$\Rightarrow m(\overset{\frown}{RS}) = 2m(\angle RPS)$$
$$= 2(54^0)$$
$$= 108^0$$

Thus $m(\angle ROS)=m(\overset{\frown}{RS})=108°$.

Theorem 5.2: In a circle, inscribed angles subtended by the same arc are congruent.

Proof: Given circle O, inscribed angles B and D subtended by the same arc AC.

We want to show that: m(∠ABC)=m(∠ADC)

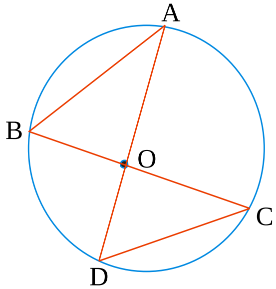


Figure 5.33

Statements	Reasons
1. m(∠ABC)= $\frac{1}{2}m(\overset{\frown}{AC})$	1. Theorem 5.1
2. m(∠ADC) = $\frac{1}{2}m(\overset{\frown}{AC})$	2. Theorem 5.1
3. m(∠ABC)=m(∠ADC)	3. Substitution

Examples 7: In Figure 5.34 to the right,

m(∠CPD) =120^0, m(∠PCD) = 30^0

find m(∠A).

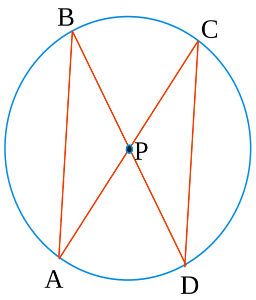


Figure 5.34

Solution:

m(∠CPD)=120° .... Given

m(∠PCD)=30°............ Given

m(∠CPD)+m(∠PCD)+m(CDP)=180° … why?

120°+30°+m (∠CDP)=180°

m(∠CDP)=180°-150°

=30°

Therefore, m(∠CDP)=30°.

Hence m(∠CDB)=m(∠CAB)=30°.................. Theorem 5.2

Exercise 5B

1. In Figure 5.35 to the right, O is the center of the circle. If m(∠ABC)=30^0, $\overline{CB} // \overline{OA}$ and $\overline{CO}$ *and* $\overline{AB}$ intersect at D, find m(∠ADC).

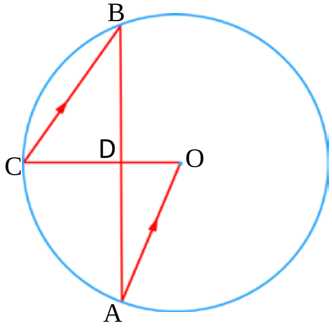


Figure 5.35

2. In Figure 5.36 to the right, if m($\overset{\frown}{AHB}$) = 100^o and m($\overset{\frown}{DIC}$) = 80^o m(∠BAC) = 50° and $\overleftrightarrow{FG}$ is tangent to the circles at C, then find each of the following.

   a. m(∠BDC)
   b. m(∠ACD)
   c. m(∠AEB)

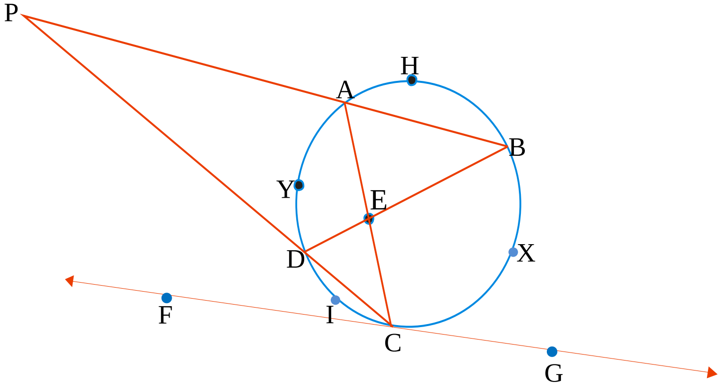


Figure 5.36

3. In Figure 5.37 below, O is the centre of the circle $\overrightarrow{PA}$ *and* $\overrightarrow{PB}$ are tangents to the circle at A and B, respectively. If m(∠ACB)=115^0 ,then find:

   a. m($\overset{\frown}{AB}$)

   b. m($\overset{\frown}{ACB}$)

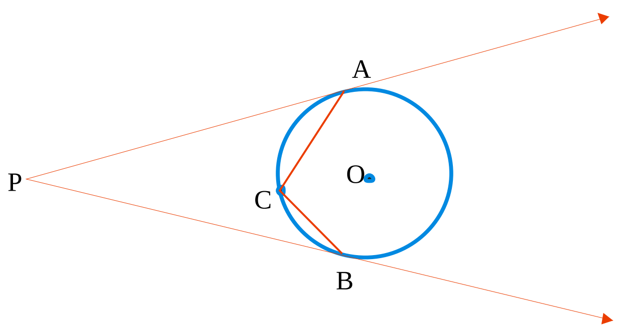


Figure 5.37

4. In Figure 5.38 to the right, O is the center of the circle. If m(∠B)=140⁰, What is m(∠AMC)?

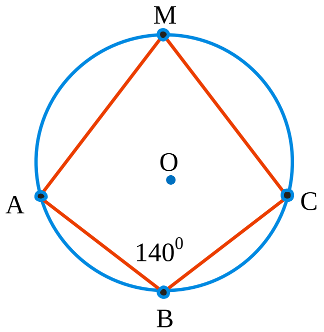


Figure 5.38

Challenge Problems

5. In Figure 5.39 to the right, O is the center of the circle, m(∠ABC) = 80^0 and m(∠AED) = 20^0 then what is m(∠DOC)?

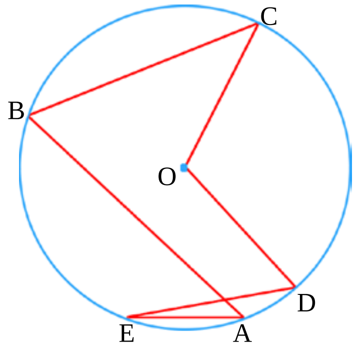


Figure 5.39

6. In Figure 5.40 to the right, O is the center of the circle and m $(\overset{\frown}{BYC}) = 40^0$, $m(\overset{\frown}{AXD})=120^0$ and m(∠ADB) $=50^0$. What is m(∠DOC) and $m(\overset{\frown}{DC})$?

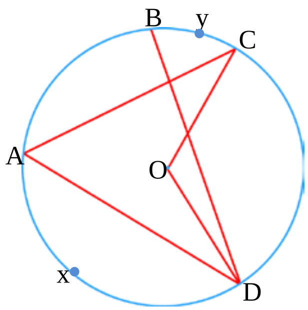


Figure 5.40

7. In Figure 5.41 given to the right , O is the center of the circle and m(∠BOC) = 120^0. What is m(∠ADC)?

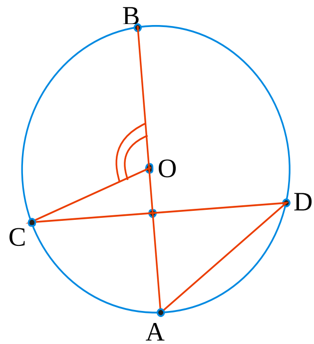


Figure 5.41

5.2.3 Angles Formed by Two Intersecting Chords

Activity 5.2

Discuss with your friends.

1. In Figure 5.42 given to the right, find m(∠x).

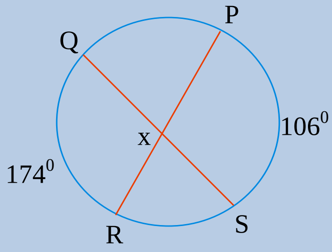


Figure 5.42

2. In Figure 5.43 given to the right, can you derived a formula m(∠a) and m(∠b)

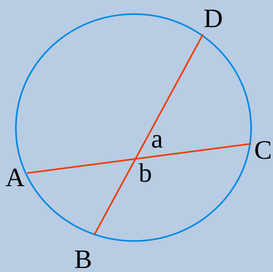


Figure 5.43

Theorem 5.3: The measure of an angle formed by two chords intersecting inside a circle is half the sum of the measures of the arcs subtending the angle and its vertical opposite angle.

Proof: Given $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersecting at P inside a circle

We want to show that:

$$m(\angle BPD) = \frac{1}{2} m(\overset{\frown}{AYC}) + \frac{1}{2} m(\overset{\frown}{DXB})$$

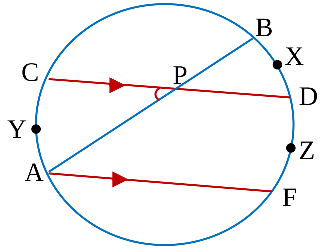


Figure 5.44

Statements	Reasons
1. Draw a line through A such that $\overline{AF} \parallel \overline{CD}$	1. construction
2. m(∠ BPD) = m (∠BAF)	2. corresponding angles formed by two parallel lines and a transversal line
3. m (∠ BAF) = $\frac{1}{2}$ m ($\overset{\frown}{BDF}$)	3. Theorem 5.1
4. m (∠ $\overset{\frown}{AYC}$) = m ($\overset{\frown}{DZF}$)	4. Why ?
5. m (∠ BPD) = $\frac{1}{2}$ m ($\overset{\frown}{BDF}$)	5. Why?
6. m(∠BPD) = $\frac{1}{2}$ m ($\overset{\frown}{BXD}$)+ m $\frac{1}{2}$ ($\overset{\frown}{DZF}$)	6. Why?
7. m (∠BPD)= $\frac{1}{2}$ m ($\overset{\frown}{AYC}$) + $\frac{1}{2}$ m ($\overset{\frown}{BXD}$)	7. Substitution

Examples 8: In Figure 5.45 given to the right, find the value of β, if m ($\overset{\frown}{AB}$) = 82° and m ($\overset{\frown}{DC}$) = 46°

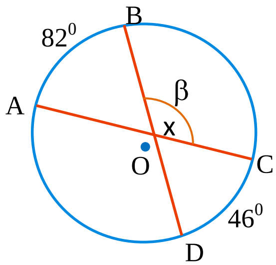


Figure 5.45

Solution:

To find β, simply by theorem 5.3

m (∠AXB) = $\frac{1}{2}$ ($\overset{\frown}{AB}$ + $\overset{\frown}{DC}$)

= $\frac{1}{2}$ (82° + 46°)

= $\frac{1}{2}$ (128°) = 64°

m (∠AXC) = 180°. . . . . . (Why)?

m (∠AXC) = m (∠AXB) + m (∠BXC)

180° = 64° + m(∠BXC)

116° = m(∠BXC)

Therefore, β = 116°

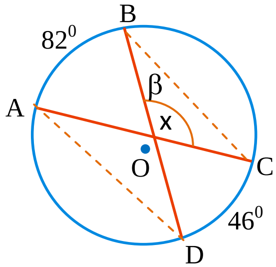


Figure 5.46

Examples 9: In Figure 5.47 given to the right, find the value of a and b.

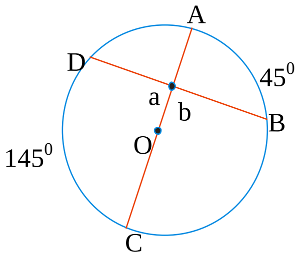


Figure 5.47

Solution:

$m(\angle a) = \frac{1}{2}[m(\overset{\frown}{DC}) + m(\overset{\frown}{AB})]$ .... Theorem 5.3

$= \frac{1}{2}(145^0 + 45^0)$

$= \frac{1}{2}(190^0)$

$= 95^0$ and

$m(\angle a) + m(\angle b) = 180^0$ .... Angle sum theorem

$95^0 + m(\angle b) = 180^0$

$m(\angle b) = 85^0$

Exercise 5C

1. Find the values of the variables in Figure 5.48 to the right.

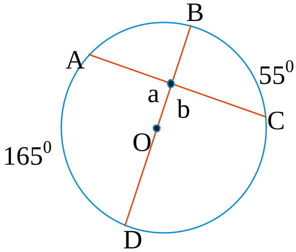


Figure 5.48

2. Find the values of the variables in Figure 5.49 below.

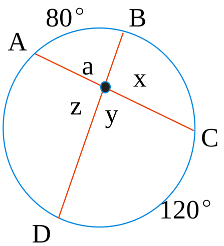


Figure 5.49

3. Find the $m(\overset{\frown}{AB})$ and $m(\overset{\frown}{DC})$ in Figure 5.50 to the right.

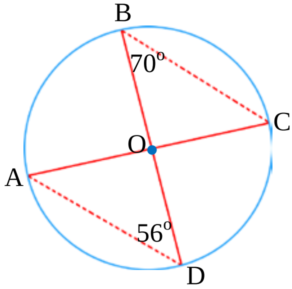


Figure 5.50

5.2.4 Cyclic Quadrilaterals

Group Work 5.3

Discuss with your friends/ partners.

1. What is a cyclic quadrilateral?
2. Based on Figure 5.51 answer the following questions.
   a. What is the sum of the measure angles A and C?
   b. What is the sum of the measure angles B and D?
   c. Is ABCD is a cyclic quadrilateral?

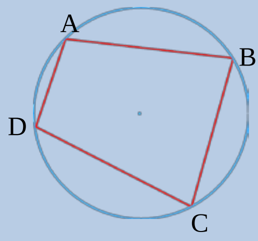


Figure 5.51

3. Find the sizes of the other three angles in the cyclic quadrilateral, if AB // DC.

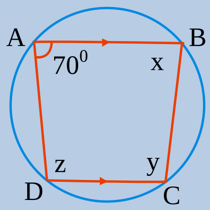


Figure 5.52

4. Angle $\angle ECD = 80^0$. Explain why AEDB is a cyclic quadrilateral. Calculate the size of angle $\angle EDA$.

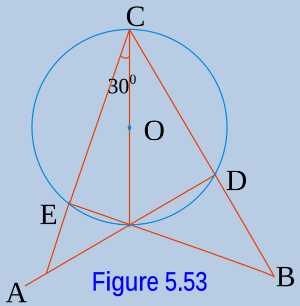


Figure 5.53

5. In Figure 5.54 below ABOD is acyclic quadrilateral and O is the center of the circle. Find x ,y, z and w, if DO // AB.

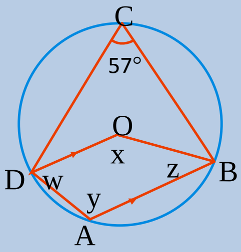


Figure 5.54

Draw a circle and mark four points A, B, C and D on it. Draw quadrilateral ABCD as shown in Figure 5.55. This quadrilateral has been given a special name called **cyclic quadrilateral.**

Definition 5.7: A quadrilateral inscribed in a circles is called cyclic quadrilateral.

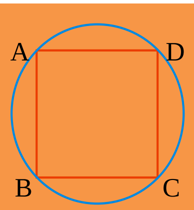


Figure 5.55 ABCD is cyclic quadrilateral

Property of Cyclic Quadrilateral

i. In Figure 5.55 above $m(\angle A)+m(\angle c)=180°$.

ii. Similarly $m(\angle B)+m(\angle D)=180°$.

Theorem 5.4: In cyclic quadrilateral, opposite angles are supplementary.

Given: ABCD is a quadrilateral inscribed in a circle.

We want to show that:

i. $m(\angle A)+m(\angle C)=180^0$

ii. $m(\angle B)+m(\angle D)=180^0$

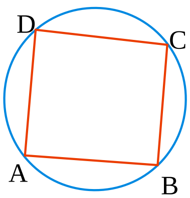


Figure 5.56

Proof

Statements	Reasons
1. $\angle DAB$ and $\angle DCB$ are opposite angles	1. Given
2. $m(\angle DAB)=\frac{1}{2}m(\overset{\frown}{DCB})$ and $m(\angle DCB)=\frac{1}{2}m(\overset{\frown}{DAB})$	2. Theorem 5.1

3. $m(\angle DAB)+m(\angle DCB)=\frac{1}{2}[m(\overset{\frown}{DCB})+m(\overset{\frown}{DAB})]$	3. By addition property
4. $m(\angle DAB)+m(\angle DCB)=\frac{1}{2}(360^0)$	4. Degree measure of a circle
5. $m(\angle A)+m(\angle C)=180^0$	5. Supplementary

? **Can you prove similarly $m(\angle B)+m(\angle D)=180^0$?**

Examples 10: In Figure 5.57 to the right, ABCD is a cyclic quadrilateral E is on $\overline{CD}$. If $m(\angle C)=110^0$, find x and y.

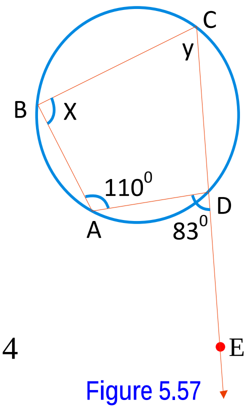


Figure 5.57

Solution:

$m(\angle BAD + m(\angle BCD)=180^0$ ....Theorem 5.4

$m(\angle y) + 110^0 = 180^0$

Therefore, $y=70^0$

$m(\angle ADE) + m(\angle CDA) = 180^0$ ....straight angle and $\overrightarrow{CE}$ is a ray.

$83^0 + m(\angle CDA) = 180^0$

Therefore, $m(\angle CDA)=97^0$

$m(\angle CBA) + m(\angle CDA) = 180^0$ ....Theorem 5.4

$m(\angle x) + 97^0=180^0$

Therefore, $x=83^0$

Examples 11: ABCD is an inscribed quadrilateral as shown in Figure 5.58. Find the $m(\angle BAD)$ and $m(\angle BCD)$.

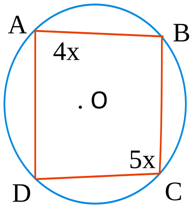


Figure 5.58

Solution:

$m(\angle DAB) + m(\angle DCB)=180^0$ .... Theorem 5.4

$4x+5x = 180^0$ .... Substitution

$9x = 180^0$

$x = 20^0$

Hence $m(\angle DAB) = 4(20^0) = 80^0$ and $m(\angle DCB) = 5(20^0) = 100^0$.

Exercise 5D

1. In Figure 5.59 to the right, A, B, C, D and E are points on the circle. If $m(<A)=100^0$, find:
   a. $m(\angle C)$
   b. $m(\angle D)$

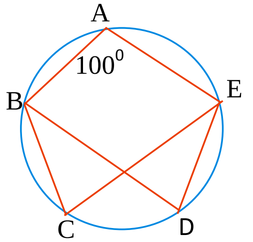


Figure 5.59

2. In Figure 5.60 to the right, ABCD is a cyclic trapezium where $\overline{AB} // \overline{CD}$. If $m(\angle A) = 95^0$, then find the measure of the other three angles.

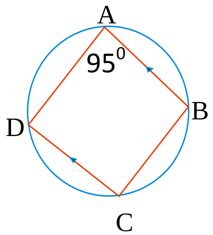


Figure 5.60

3. Consider the quadrilateral ABCD. Is it a cyclic quadrilateral?

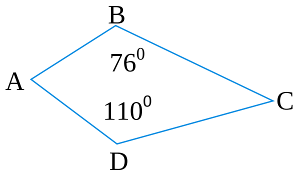


Figure 5.61

4. In Figure 5.62 to the right, ABCD is an inscribed quadrilateral. Find the measure of $\angle BAD$ and $\angle BCD$.

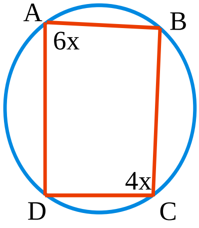


Figure 5.62

Challenge Problems

5. In Figure 5.63 to the right, find
   a. m(∠ABC)
   b. m(∠ADC)
   c. m(∠PAB)

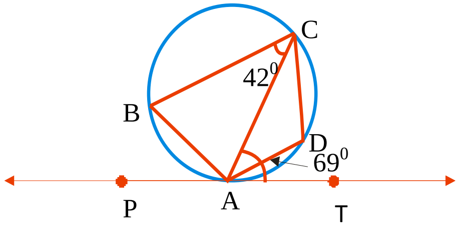


Figure 5.63

Summary for Unit 5

1. *A circle is the set of all points on a plane that are equidistant from a given point, called the **centre of the circle.***
2. *A **Chord** of a circle is a segment whose end points are on the circle.*
3. *A **diameter** of a circle is any chord that passes through the center and denoted by d.*
4. *A **radius** of a circle is a segment that has the center as one end point and a point on the circle as the other end point, and denoted by r.*
5. *The perimeter of a circle is called its **circumference.***
6. ***An arc** is part of the circumference of a circle.*
   Arcs are classified in the following three ways. .
   a. ***Semi-circle**: an arc whose end points are also end points of a diameter of a circle.*
   b. ***Minor arc:** is the part of a circle less than a semi circle.*
   c. ***Major arc:** is the part of a circle greater than a semi-circle.*
7. ***A sector** of a circle is the region bounded by two radii and an arc of the circle.*
8. ***A segment:** of a circle is the region bounded by a chord and the arc of the circle.*
9. ***A central angle** of a circle is an angle whose vertex is the center of the circle and whose sides are radii of the circle.*
10. ***An inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle.*

11. *A quadrilateral inscribed in a circles is called* ***cyclic quadrilateral.***
12. *The measure of an inscribed angle is equal to one half of the measure of its intercepted arc.*
13. *The measure of an angle formed by two chords intersecting inside a circle is half the sum of the measure of the arcs subtending the angle and its vertical opposite angle.*
14. *The sum of the opposite angles of cyclic quadrilateral is* ***supplementary***

Miscellaneous Exercise 5

I. Write true for the correct statements and false for the incorrect ones.

1. Opposite angles of an inscribed quadrilateral are supplementary.
2. A central angle is not measured by its intercepted arc.
3. An angle inscribed in the same or equal arcs are equal.
4. A tangent to a circle can pass through the center of the circle.
5. If the measure of the central angle is double, then the length of the intercepted arc is also double.

II. Choose the correct answer from the given four alternatives

6. In Figure 5.64 to the right, O is the center of the circle. What is the value of x?

   a. 36^0　　c. 10^0
   b. 60^0　　d. 18^0

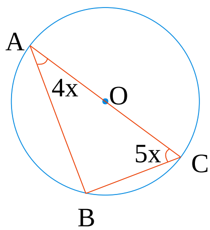


Figure 5.64

7. In Figure 5.65 below, $\overline{OA}$ and $\overline{OB}$ are radii of circle O. Which of the following statement is true?

   a. AB = OA
   b. AB>OA
   c. AB<OA
   d. None

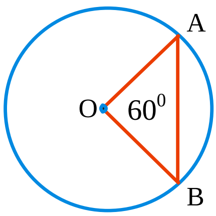


Figure 5.65

8. The measure of the opposite angles of a cyclic quadrilateral are in the ratio 2:3. What is the measure of the largest of these angles?

   a. 27^0 b. 120^0 c. 60^0 d. 108^0

III. Workout problems

9. In Figure 5.66 to the right, lines $\overleftrightarrow{AB}$ *and* $\overleftrightarrow{CD}$ are parallel and $m(\overset{\frown}{AB}) = 100^0$ and $m(\overset{\frown}{CD}) = 80^0$. What is $m(\angle AED)$?

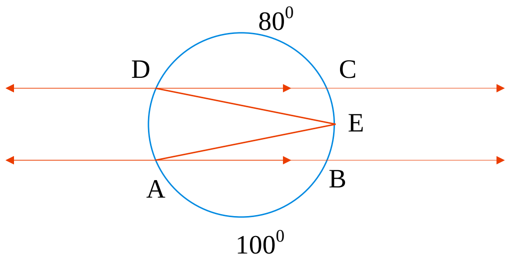


Figure 5.66

10. In Figure 5.67 below, find the value of the measure $\angle a$.

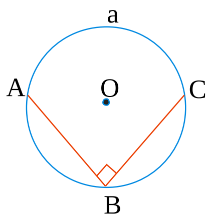


Figure 5.67

11. Construct the circle through A, B and C where AB=9cm, AC = 4cm and BC = 4cm.

12. In Figure 5.68 given below;

    a. If $m(\angle AOC) = 140^0$, find $m(\angle ABC)$ and $m(\angle ADC)$.

    b. If $m(\angle ABC) = 60^0$, find $m(\angle AOC)$ and $m(\angle ADC)$.

    c. If $m(\angle AOC) = 200^0$ find $m(\angle ABC)$.

    d. If $m(\angle ABC) = 80^0$, find $m(\angle OAC)$.

    e. If $m(\angle OCA) = 20^0$, find $m(\angle ADC)$.

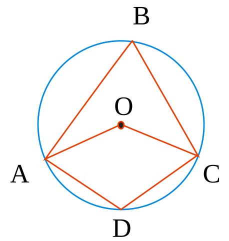


Figure 5.68

13. ABCD is a quadrilateral inscribed in a circle BC = CD. AB is parallel to DC and $m(\angle DBC) = 50^0$. Find $m(\angle ADB)$.

14. O is the center of the circle.

    Calculate the size of angle $\angle QSR$

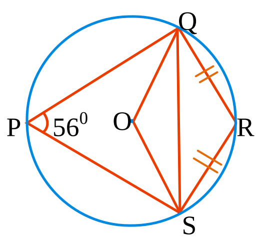


Figure 5.69

15. For the given circle O is its center and the two secant lines m and n are parallel. Find x.

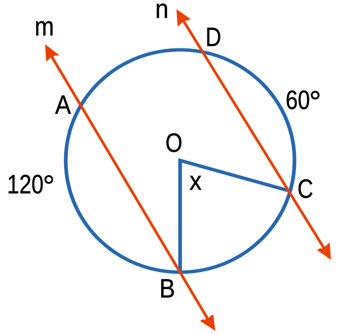


Figure 5.70

16. In Figure 5.71 below $\overline{PT}$ is a chord and O is the center of the circle. Calculate the size of m (∠TPO).

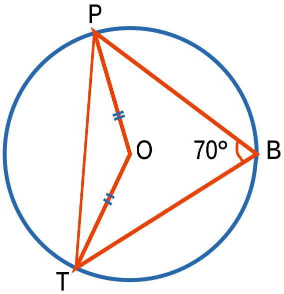


Figure 5.71

17. O is the center in Figure 5.72 to the right, Angle ABC = 158°.
Find (a) reflex angle AOC (c) angle ADC
(b) angle AOC

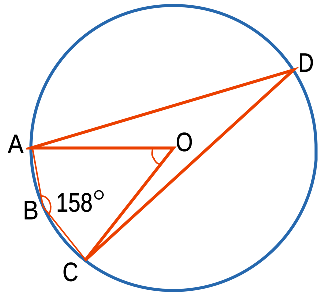


Figure 5.72

18. Find all the unknown angles in the figure in which $\overline{AB}$ // $\overline{DC}$ and angle ACD = 24°.

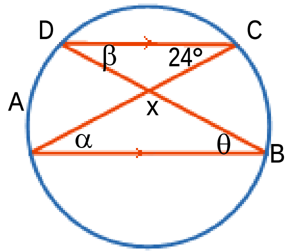


Figure 5.73

19. Given that angle BFC = 58°.
Angle BXG = 48° and angle CBF = 22°.
Find (i) ∠BGX (iv) ∠BCG
(ii) ∠BGF (iv) ∠BFG
(iii) ∠BCF

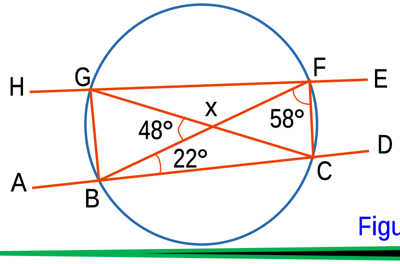


Figure 5.74

UNIT 6 INTRODUCTION TO PROBABILITY

Unit outcomes

After Completing this unit, you should be able to:

- understand the concept of certain, uncertain and impossible outcomes.
- know specific facts about event, sample space and probability of simple events.

Introduction

When you buy a lottery ticket you cannot be 100% sure to win. Some things can occur by chance or things what you expected may not occur at all. The occurrence or non-occurrence of these things are studied in mathematics by the theory of probability. So in this unit you will learn the simple and introductory concepts of probability.

Figure 6.1

This morning there is a chance of heavy rain with the possibility of thunder. In the afternoon the rain will die away and it is likely that the sun will break through the clouds, probably towards evening.

Weather forecasts are made by studying weather data and using a branch of mathematics called **probability**.

Probability uses numbers to represent how likely or unlikely it is that an event such as 'a thunderstorm' will happen.

Probability is used by governments, scientists, economists, medical researchers and many other people to predict what is likely to happen in the future by studying what has already happened.

Group work 6.1

Discuss with your group.

1. **Find out your classmates' favorite and least favorite school subjects. Ask them what they like and dislike about each.**
2. **Also find out if they like or dislike going to school overall. Find out why or why not.**
3. **Make a chart presenting your findings to the class.**

6.1 The Concept of Probability

Consider a two sided coin. For convenience let the two faces (sides) of the coin be called head (H) and tail (T). If we toss (flip) the coin, the experience shows that the possible outcomes are H (head) or T (tail) but not both. Similarly discuss example 1 with your group.

Example 1

a. Tossing a coin.
b. Tossing two coins.
c. Tossing three coins.
d. Tossing four coins.
e. Rolling a die.

Figure 6.2

Definition 6.1: *A process* of an observation is often called an experiment.

Definition 6.2: The set of all possible out comes of an experiment is called a sample space/the possibility set/ of the experiment and denoted by S.

Definition 6.3: A subset of the sample space of an experiment is called an event and denoted by E.

To illustrate the above definition /explanation/ consider the following summary of probability experiments.

Experiment	Sample space
a. Toss a coin.	{Head, tail} ={H, T}
b. Toss two coins.	{HH, HT, TH, TT}
c. Toss three coins.	{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
d. Roll a die.	{1, 2, 3, 4, 5, 6}
e. Choose an English vowel.	{a, e, i, o, u}
f. Answer a true – false question.	{True, False}

a) Showing head

b) Showing tail

Figure 6.3 coins

Example2: suppose an experiment is rolling a die.

a. List the elements of the sample spaces.

b. List the elements of the set of the event "the number shown is prime".

c. List the elements of the set of the event "the number shown is odd".

d. List the elements of the set of the event "the number shown is even".

Solution

a. The sample space are $\{1, 2, 3, 4, 5, 6\} = S$

b. The prime number is $\{2, 3, 5,\}$ = E

c. The odd number is $\{1, 3, 5,\}$ = E

d. The even number is $\{2, 4, 6,\}$ = E

Figure 6.4 Die

Activity 6.1

1. Identify the following events has certain or impossible events.

a. You will grow to be 30 centimeters tall.

b. You will live to be 240 years old.

c. You will die.

d. A newly born baby will be a girl.

2. Give two examples of events that you think

a. are impossible.

b. are certain.

3. Locate each of the following situations on the probability scale.

a. You will have match home work to night.

b. Ababy born today was a girl.

c. The local meteorologist predicts a 40% chance that it will rain tomorrow.

d. If will snow in your town in August.

Definition 6.4: If the probability of an event is one, then it is called certain event.

Example 3: a. Night will follow day.

b. December following November next year.

c. The next person to come into the room will be right handed.

Definition 6.5: If the probability of an event is 0, then it is called *impossible out comes.*

Example 4: a. When water boils it changes to milk.

b. Two lines intersect at four points.

c. You will grow to be 50 meters tall.

d. It will rain to morrow.

Definition 6.6: All probabilities must have a value greater than or equal to 0 and less than or equal to 1 or $0 \le P(E) \le 1$. This can be shown on a probability scale:

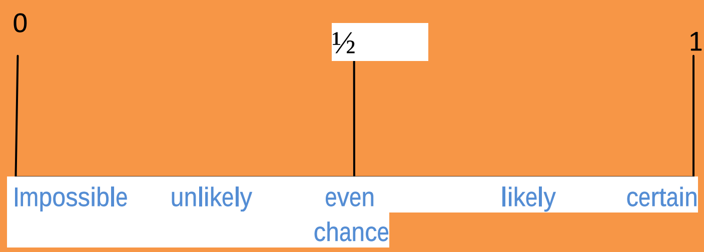


Figure 6.5 Probability scale

Exercise 6A

1. Draw a 0 to 1 probability scale and mark on it the probability that:

   a. the sea will disapper.

   b. you will buy a new pair of shoes soon.

   c. the sun will not rise next week.

d. a member of the class will be late tomorrow.

e. a newely born baby will be a boy.

f. you will watch TV sometime to night.

g. a coin thrown in the air will land heads up.

h. a coin thrown in the air will land tails up.

2. Given two examples of events that you think

a. are impossible

b. are unlikely

c. have about an even chance

d. are likely

e. are certain

6.2 Probability of Simple Events

Group work 6.2

1. A fair six sided die is rolled. What is the probability of getting.

a. a number 4

b. an odd number

c. a multiple of 3

d. less than 5

e. more than 6

f. a number 1 or a number 2

g. an even number

h. 3 or more

i. a prime number

Figure 6.6 die

2. Suppose an experiment is rolling a die:

a. List the elements of the sample spaces.

b. List the elements of the set of the event "the sum of the number is prime.

c. List the elements of the set of the event "the sum of the number is even".

d. List the elements of the set of the event "the sum of the number is 12".

e. List the elements of the set of the event "the sum of the number is 9".

3. Nine playing cards are numbered 2 to 10. A card is selected from them at random. Calculate the probability that the card will be

a. an odd number.
b. a multiple of 4.

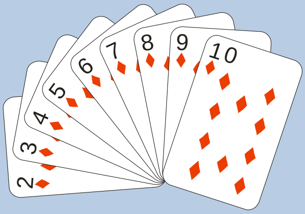


Figure 6.7 Playing Cards

4. What is the probability of an event that is certain occur?
5. What is the probability of an event that cannot occur?

Historical Note

The first book written on the subject of probability was the Book on Games of Chance by Jerome Cardano. He was an Italian physician and mathematician who lived in the 16th century.

Figure 6.8 Jerome Cardan

Definition 6.7: (probability of an event)

In words: The probability of an event is the ratio of the number of successful out comes in the event to the total number of possible out comes in the sample space.

In symbols: P (event) = $\dfrac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$

or P(E) = $\dfrac{n(E)}{n(S)}$.

Assuming that the out comes are all equally likely.

Example 5: Find the probability of getting a number 5 or 6 when a fair die is rolled.

Solution:

These are two successful out comes: 5 or 6 and there are six possible out comes: 1, 2, 3, 4, 5, 6 or n(E) = 2 and n (S) = 6.

Thus P(5 or 6) = $\frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{2}{6} = \frac{1}{3}$

> **Note:** The word fair means that each number has an equal chance of turning up: the outcomes are equally likely.

Example 6: Find the probability of drawing a card with a prime number on it from a deck of cards numbered l to 20.

Solution:

The sample spaces are $\begin{Bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \end{Bmatrix}$

The events are $\{2, 3, 5, 7, 11, 13, 17, 19\}$.

Hence P(prime number) = $\frac{\text{number of successful out comes}}{\text{total number or possible out comes}} = \frac{8}{20} = \frac{2}{5}$

Example 7: A bag contains 8 discs of which 4 are red, 3 are blue and 1 is yellow. Calculate the probability that when one disc is drawn from the bag it will be

a. red c. blue

b. yellow d. yellow or blue

Figure 6.9

Solution: There are 8 discs altogether so the total number of possible out comes is 8

a. P (red) = $\frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{4}{8} = \frac{1}{2}$

b. P(yellow) = $\frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{1}{8}$

c. P(blue) = $\frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{3}{8}$

d. P(yellow or blue) = $\frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{4}{8} = \frac{1}{2}$

Example 8: You randomly draw a slip of paper from a box containing 4 slips. A red slip, a black slip, a white slip and a pink slip.

a. How many possible outcomes are there?

b. What is the probability of each outcomes?

Solution: a. There are 4 possible outcomes.

b. Since each slip is equality likely to be drawn, the probability of each outcomes is $\frac{1}{4}$.

Example 9: A letter is chosen at random from the word PROBABILITY. Work out the probability that it will be:

a. R b. Y c. B d. I or A e. B or I

Solution: Total number of possible outcomes is 11.

a. $P(R) = \frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{1}{11}$

b. $P(Y) = \frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{1}{11}$

c. $P(B) = \frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{2}{11}$

d. $P(I \text{ or } A) = \frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{3}{11}$

e. $P(B \text{ or } I) = \frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{4}{11}$

Note: An outcome is said to occurre at random if each out comes is equally likely to occur.

Example 10: Six slips of paper one labeled with the letters of "POTATO". The slips are shuffled in a hat and you randomly draw one slip. What is the probability that the slip you draw:

a. the letter T?

b. either the letter O or the letter A?

c. the letter Z?

d. one of the letters P, O. T or A?

Solution: a. $P(t)=\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}=\frac{2}{6}=\frac{1}{3}$

b. $P(O,A)=\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}=\frac{3}{6}=\frac{1}{2}$

c. $P(Z)=\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}=\frac{0}{6}=0$

d. $P(P, O, T \text{ or } A)=\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}=\frac{6}{6}=1$

Example 11: A bowl of fruit contains 3 apples, 4 bananas, 2 lemon and 1 orange. Abebe takes one piece of fruit without looking. What is the probability that he takes.

a. an apple c. a lemon

b. a banana d. an orange

Write each answer in three ways, that means

i. as a fraction

ii. as a decimal and

iii. as a percentage

Figure 6.10

Solution: A bowl of fruit contains = 3 apples + 4 bananas + 2 lemon + 1 orange = 10

a. $P(\text{Lemon})=\frac{\text{number of successful out comes}}{\text{total number of possible out comes}}=\frac{3}{10}$.

Therefore i. probability of an apple as a fraction is $\frac{3}{10}$.

ii. probability of an apple as a decimal is 0.3.

iii. probability of an apple as a percentage is $\frac{3}{10}\times 100\% = 30\%$.

b. $P(\text{banana})=\frac{\text{number of successful out comes}}{\text{total number of possible out comes}}=\frac{4}{10}$.

Therefore, i. probability of an apple as a fraction is $\frac{4}{10}$.

ii. probability of an apple as a decimal is 0.4.

iii. probability of an apple as a percentage is 40%.

c. $P(\text{lemon})=\frac{\text{number of successful out comes}}{\text{total number of possible out comes}}=\frac{2}{10}=\frac{1}{5}$.

Therefore, i. probability of a lemon as a fraction is $\frac{1}{5}$.

ii. probability of a lemon as a decimal is 0.2.

iii. probability of a lemon as percentage is 20%.

d. P(an orange) = $\frac{\text{number of successful out comes}}{\text{total number of possible out ocmes}} = \frac{1}{10}$.

Therefore , i. probability of an orange as a fraction is $\frac{1}{10}$.

ii. probability of an orange as a decimal is 0.1.

iii. probability of an orange as a percentage is 10%.

Example 12: Two dice are rolled. State the probability of each event.

a. The sum is 7.

b. The sum is 13.

c. The sum is less than 13.

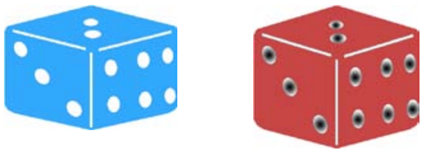


Figure 6.11 Dice

Solution: There are 6 numbers on each die. The sample space has 36 or 6^2 out comes.

Sample space

	Second die					
First Die	**1**	**2**	**3**	**4**	**5**	**6**
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

a. The out comes with sum 7 are:

{(1, 6), (2, 5), (3, 4), (4, 3) (5, 2), (6, 1)}.

P(sum =7) = $\frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{6}{36} = \frac{1}{6}$.

b. There are 0 out comes with sum 13.

$\Rightarrow$ P(sum = 13) = $\frac{0}{36} = 0$.

c. All 36 out comes have a sum less than 13.

P(sum < 13) = $\frac{36}{36} = 1$.

Exercise 6B

1. A counting number less than 30 is chosen at random. What is the probability that the number chosen:
   a. is a multiple of 4?
   b. is a square?
   c. is a cube number?
   d. is a prime?

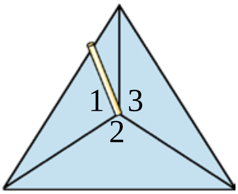
2. A jar contains 2 orange, 5 blue, 3 red and 4 yellow marbles. A marble is drawn at random from the jar. Find each probability.
   a. p (orange)
   b. p (red)
   c. p (blue)
   d. p (green)

Figure 6.12

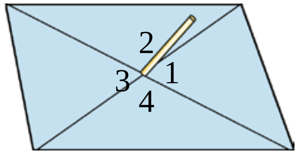
3. Two dice are thrown at the same time. State the probability of each event:
   a. The sum is 5
   b. The sum is 9
   c. The sum is 12

4. A game is played with two spinners. You multiply the two numbers on the spinners land to get the score

Spinner A



Spinner B



This score is 2×4 = 8

Figure 6.13 Spinner

   a. Copy and complete the table to show all the possible scores. One score has been done for you.

Spinner B

Spinner A

×	1	2	3	4
1				
2				8
3				

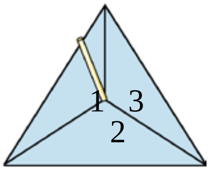
b. Work out the probability of getting a score of 6.

c. Work out the probability of getting a score that is an odd number.

5. This spinner is spun. What is the probability of getting:

a. a number 1

b. an odd number



The three-sided spinner has landed on two

Figure 6.14

6. Nine counters numbered 2 to 10 are put in a bag. One counter is selected at random.

What is the probability of getting a counter with

a. a number 5

b. an odd number

c. a prime number

d. a perfect square number

e. a multiple of 3?

7. A hundred raffle tickets are sold. Raman buys 8 tickets, Susan 5 tickets and Aster 12 tickets. What is the probability that the first prize will be won by

a. one of these three

b. Raman

c. Susan

d. Somebody other than Aster?

Write each answer in three ways:

a. as a fraction

b. as a decimal and

c. as a percentage

Challenge Problem

8. When three dice are thrown at the same time what is the probability that the sum of the number of dots on the top faces will be 6?

9. A number is selected at random from 1 to 100. State the probability that:

a. The number is odd

b. the number is divisible by 5

c. the number is even or divisible by 5

d. the number is divisible by 5 or 3.

Summary for Unit 6

1. *A process of an observation is often called an experiment.*
2. *The set of all possible outcomes of an experiment is called a sample space /the possibility set/ of the experiment and denoted by S.*
3. *A subset of the sample space of an experiment is called an event and denoted by E.*
4. *An event which is certain to happen has a probability of 1.*
5. *An event which cannot happen is (impossible) outcomes has a probability of 0.*
6. *The probability that an event will happen is calculated by:*

   $$\text{Probability of an event} = \frac{number\ of\ successful\ out\ comes}{total\ number\ of\ possible\ out\ comes}.$$
7. *The probability of an event happening is always greater than or equal to 0 (impossible) and less than or equal to 1 (certain). This can be written as* $0 \leq \text{probability an event} \leq 1$.

Miscellaneous Exercise 6

I. Write true for the correct statements and false for the incorrect one

1. The probability of an event that is certain to occur is 1.
2. The probability of an event that is an impossible out comes to occur is 0.
3. The probability of getting a sum of 7 or 11 by rolling two dice is $\frac{2}{9}$.
4. If the set of all possible outcomes is equal to an event then the probability of an event is 1.
5. Suppose that two dice are tossed, and then the probability of the sum 1 is also 1.

II. Choose the correct answer from the given alternatives

6. If 2 fair dice are tossed, what is the probability that the sum of the number of dots on the top faces will be 7?

   a. $\frac{1}{9}$ c. $\frac{1}{6}$
   b. $\frac{5}{36}$ d. $\frac{7}{36}$

7. A bag contains 6 white balls, 4 red balls and 5 black balls. If a ball is drawn from the bag at a random, then which of the following is true.
   a. Probability of a white ball is $\frac{2}{5}$.
   b. Probability of red or a black ball is $\frac{3}{5}$.
   c. Probability of not getting a black ball is $\frac{2}{3}$.
   d. All are true

8. A pairs of fair dice is tossed. What is the probability of not getting a sum 5 or 9?
   a. $\frac{2}{9}$ b. $\frac{8}{9}$ c. $\frac{7}{9}$ d. $\frac{1}{9}$
9. Which of the following is true about a probability scale?
   a. Probability of unlikely between 0 and $\frac{1}{2}$.
   b. Probability of even chance is $\frac{1}{2}$.
   c. Probability of likely between $\frac{1}{2}$ and 1.
   d. All are true

III. Work out problems

10. A letter of the English alphabet is chosen at random. Calculate the probability that the letter so chosen be:
    a. vowel
    b. Precedes m and is a vowels
    c. follows m and is a vowels
11. If three coins are thrown. What is the probability of obtaining.
    a. all heads
    b. all tails
    c. at least one heads
    d. at least two heads
12. A letter is chosen at random from the words ETHIOPIA MATHS. What is the probability that the letter E is chosen?
13. One letter is chosen at random from the word ISOSCELES. What is the probability of choosing.
    a. the letter C
    b. the letter E
    c. a vowel
    d. a consonant

UNIT 7

GEOMETRY AND MEASUREMENTS

Unit outcomes

After Completing this unit, you Should be able to:

- understand basic concepts about right angled triangles.
- apply some important theorems on right angled triangles.
- know basic principles of trigonometric ratios.
- know different types of pyramid and common parts of them.

Introduction

In this unit you will in detail learn about the basic properties of right angled triangles, by using two theorems on this triangle. You will also learns about a new concept that is very important in the field of mathematics known as trigonometric ratios and their real life application to solve simple problems. In addition to this you will also learn a solid objects known as pyramids and cone and their basic parts.

7.1 Theorems on the Right Angled Triangle

In your earlier classes you have learnt many things about triangles. By now, you do have relatively efficient knowledge on some of the properties of triangles in general. In this sub topic we will give special attention to the properties of right angled triangles and the theorems related to them.

Right angled triangles have special properties as compared to other types of triangles. Due to their special nature they have interesting properties to deal with.

There are some theorems and their converses that deal with the properties of right angled triangles.

Group Work 7.1

1. Name the altitudes drawn from the right angle to the hypotenuse of the given right angled triangles.

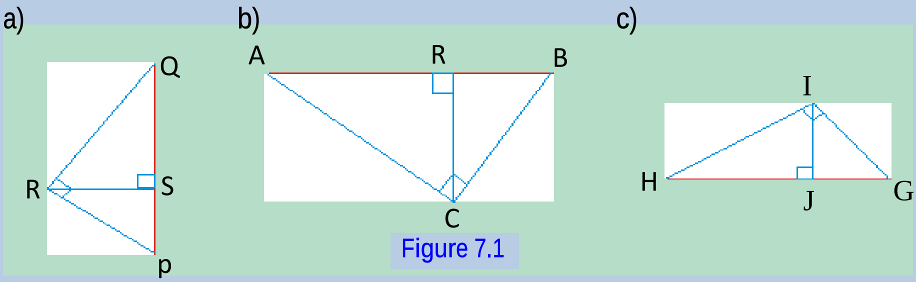


Figure 7.1

2.

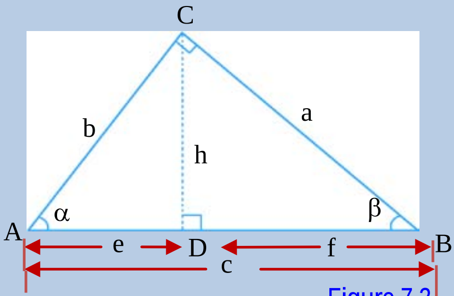


Figure 7.2

In Figure 7.2 To the left of the unknown quantities

	Δ ABC	Δ ADC	Δ BDC
Hypotenuse			
leg			
leg			

3. In Figure 7.2 above, find three similar triangles.

4. In Figure 7.2 above Δ CAB ~ Δ DAC. Why?

Figure 7.3 Euclid

Historical note

There are no known records of the exact date or place of Euclid's birth, and little is known about his personal life. Euclid is often referred to as the "**Father of Geometry**." He wrote the most enduring mathematical work of all time, the **Elements**, a13volume work. The Elements cover plane geometry, arithmetic and number theory, irrational numbers, and solid geometry.

7.1.1 Euclid's Theorem and Its Converse

In Figure 7.2 above the altitude $\overline{CD}$ of ΔABC divides the triangle in to two right angled triangles: ΔADC and ΔBDC. You can identify three right angled triangles (ΔABC, ΔADC and ΔBDC). If you consider the side correspondence of the three triangles as indicated in Table 7.1 below, it is possible to show a similarity between the triangles.

Table 7.1

	Δ ABC	**Δ ADC**	**Δ BDC**
Hypotenuse	c	b	a
leg	a	h	f
leg	b	e	h

The AA similarity theorem could be used to show that:

1. $\Delta DBC \sim \Delta DCA$
2. $\Delta ABC \sim \Delta CBD$
3. $\Delta CAB \sim \Delta DAC$

From similarity (2) you get the following proportion:

$$\frac{AB}{CB} = \frac{BC}{BD}$$

$$\Rightarrow \frac{c}{a} = \frac{a}{f}$$

$$\Rightarrow a^2 = cf$$

and from similarity (3) you get the following proportion:

$$\frac{CA}{DA} = \frac{AB}{AC}$$

$$\Rightarrow \frac{b}{e} = \frac{c}{b}$$

$$b^2 = ec$$

These relations are known as **Euclid's Theorem.**

From the above discussion, you can state the Euclid's theorem and its converse.

Theorem 7.1: (Euclid's Theorem)

In a right angled triangle with an altitude to the hypotenuse, the square of the length of each leg of the triangle is equal to the product of the hypotenuse and the length of the adjacent segment into which the altitude divides the hypotenuse:

Symbolically: 1. $(BC)^2 = AB \times BD$

Or $a^2 = c \times f$

2. $(AC)^2 = AB \times DA$

Or $b^2 = c \times e$

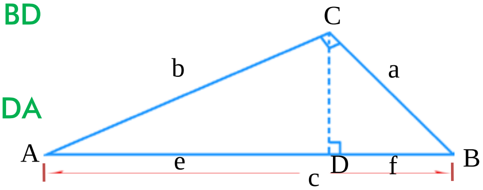


Figure 7.4

Example1: In Figure 7.5 to the right, ΔABC is a right angled triangle with $\overline{CD}$ the altitude on the hypotenuse. Determine the lengths of $\overline{AC}$ and $\overline{BC}$ if AD= 3cm and DB= 12cm.

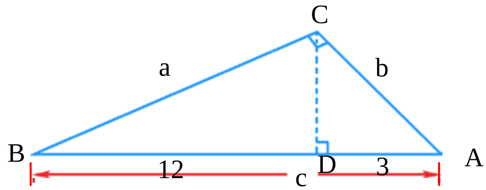


Figure 7.5

Solution:

$(AC)^2 = (AB) \times (AD)$ .... Euclid's Theorem

$(AC)^2 = (15cm) \times (3cm) = 45cm^2$… AB = BD +AD

$AC = \sqrt{45\ cm^2}$

$AC = 3\sqrt{5}cm$

$(BC)^2 = (AB) \times (BD)$ .... Euclid's Theorem

$(BC)^2 = (15cm) \times (12cm)$

$(BC) = \sqrt{180cm^2}$

$BC = 6\sqrt{5}cm$

Theorem 7.2: (Converse of Euclid's Theorem)

In a triangle if the square of each shorter side of the triangle is equal to the product of the length of the longest side of the triangle and the adjacent segment into which the altitude to the longest side divides this side, then the triangle is right angled:

Symbolically: 1. $a^2 = cf$ and

2. $b^2 = ce$ if and only if ΔABC is right angled.

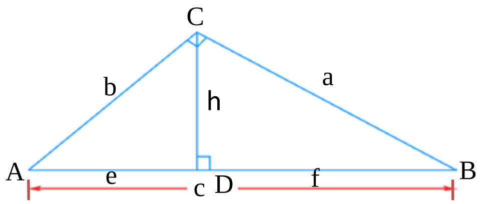


Figure 7.6

You can combine the theorem of Euclid's and its converse as follows:

ΔABC with side lengths a, b, c and h the length of the altitude to the longest side and e, f the lengths of the segments into which the altitude divide the longest side and adjacent to the sides with lengths a and b respectively is right angled if and only if $a^2 = cf$ and $b^2 = ce$.

Symbolically, ΔABC is right angled.

If and only if $a^2 = cf$ and if and only if $b^2 = ce$.

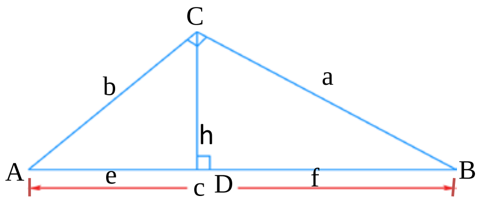


Figure 7.7

Example 2: In Figure 7.8 to the right,

$AD = 4cm$, $DB = 12cm$,

$AC = 8cm$ and $BC = 8\sqrt{3}cm$

and $m(\angle ADC) = 90^0$.

Is ΔABC a right angled?

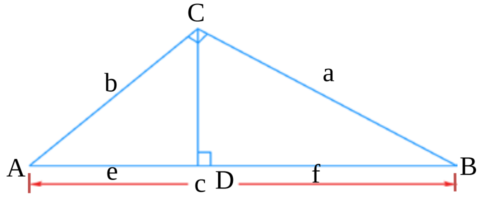


Figure 7.8

Solution:

a. $(BC)^2 = (BD) \times (BA)$ .... Theorem 7.1

$(8\sqrt{3}cm)^2 = (12cm) \times (BA)$

$192cm^2 = (12cm)(BA)$

$BA = \dfrac{192cm^2}{12cm}$

$BA = 16cm$

Thus $(AB) \times (DB) = (16cm) \times (12cm)$

$= 192cm^2$

Hence $(BC)^2 = (BD) \times (BA)$

b. $(AC)^2 = (8cm)^2 = 64cm^2 = b^2$

$(AD) \times (AB) = (16cm) \times (4cm)$

$= 64cm^2 = ce$

Hence $b^2 = ec$

Therefore from (a) and (b) and by theorem 7.2, ΔABC is a right angled triangle, where the right angle is at C.

Example 3: In Figure 7.9 below, $AC = 3\sqrt{13}cm$, $BC = 2\sqrt{13}cm$, $AB = 13cm$ and $DB = 4cm$. Is Δ ABC a right angled?

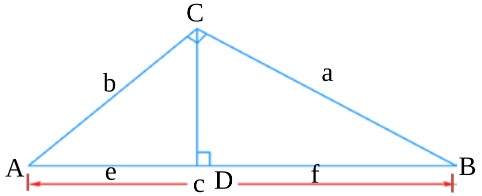


Figure 7.9

Solution:

a. $(BC)^2 = (BD) \times (AB)$ .... Theorem 7.1

Now $(BC)^2 = (2\sqrt{13}cm)^2 = 4 \times 13 = 52cm^2 = a^2$

$(AB) \times (BD) = (13cm) \times (4cm) = 52cm^2 = fc$

Therefore, $a^2 = fc$

b. $(AC)^2 = (AD) \times (AB)$ . . . . Theorem 7.1

Now $(AC)^2 = \left(3\sqrt{13cm}\right)^2 = 9 \times 13 = 117cm^2 = b^2$

$(AB) \times (AD) = 13cm \times 9cm = 117cm^2 = ec$

Therefore, $b^2 = ec$

Therefore, from (a) and (b) above and by theorem 7.2 $\triangle ABC$ is a right angled triangle.

Exercise 7A

1. In Figure 7.10 to the right, Δ ACB is a right triangle with the right angle at C and $\overline{CD} \perp \overline{AB}$ where D is on $\overline{AB}$. Find the lengths of $\overline{AC}$ and $\overline{BC}$, if AD = 6cm and DB = 12cm.

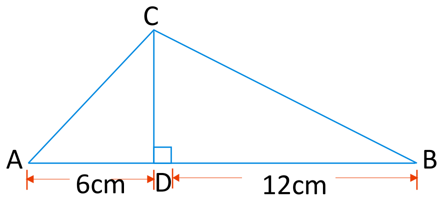


Figure 7.10

2. In Figure 7.11 below, ΔABC is a right triangle. $m(\angle ABC) = 90^0$, $\overline{BD}$ is the altitude to the hypotenuse $\overline{AC}$ of ΔABC. Find the values of the variables.

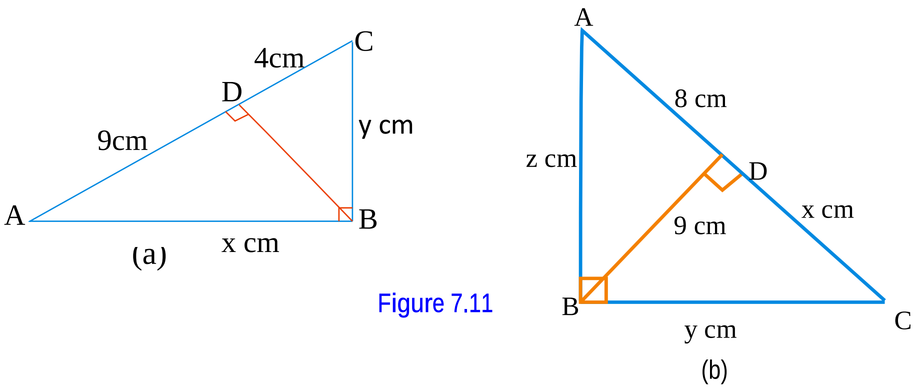


Figure 7.11

3. In Figure 7.12 to the right, ΔABC is right angled at B, $\overline{BD} \perp \overline{AC}$, BE = BC, BE = 6cm, AC = 12cm.
Find a. $\overline{BC}$
b. $\overline{DC}$
c. $\overline{AB}$

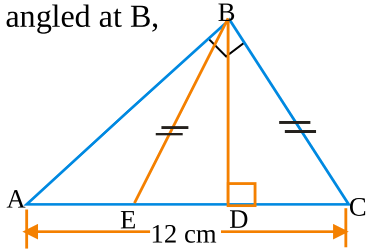


Figure 7.12

4. In Figure 7.13, AD = 3.2 cm, DB = 1.8 cm AC = 4cm and BC = 3 cm. Is ΔABC a right angled?

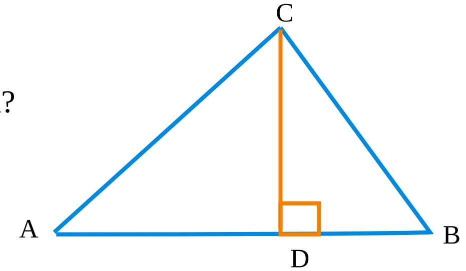


Figure 7.13

Challenge problems

5. In Figure 7.14 below, $\overparen{ABC}$ is a semicircle with center at O. $\overline{BD} \perp \overline{AC}$ such that BD = 8cm and BC = 10cm.

Find a. $\overline{CD}$
b. $\overline{AD}$
c. $\overline{AC}$
d. $\overline{OB}$

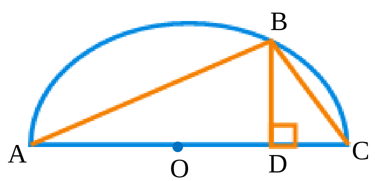


Figure 7.14

6. In Figure 7.15 below, O, is the center of the semicircle ABC. $\overline{BD} \perp \overline{AC}$, DO = 3cm and BD = 6cm. Find the radius of the circle.

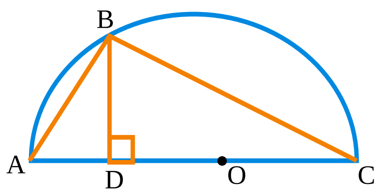


Figure 7.15

7.1.2 The Pythagoras' Theorem and Its Converse

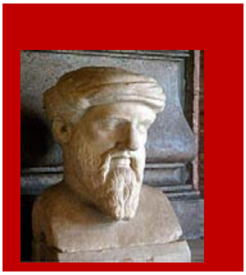


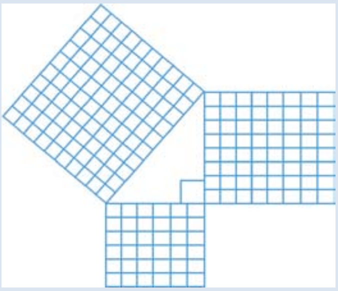
Figure 7.16 Pythagoras

Historical note

Early writers agree that Pythagoras was born on Samos the Greek island in the eastern Aegean Sea. Pythagoras was a Greek religious leader and a philosopher who made developments in astronomy, mathematics, and music theories.

Group work 7.2

1. Verify the Pythagorean property by counting the small squares in the diagrams.


(a)

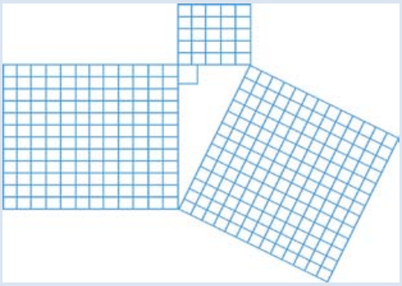

(b)

Figure 7.17

2. State whether or not a triangle with sides of the given length is a right triangle.
   a. 3m, 5m, 7m
   b. 10m, 30m, 32m
   c. 9cm, 12cm, 15cm
   d. 10cm, 24 cm, 26 cm
   e. 20mm, 21mm, 29mm
   f. 7km, 11km, 13km
3. Pythagorean triples consist of three whole numbers a, b and c which obey the rule: $a^2 + b^2 = c^2$
   a. when a = 1 and b = 2, find the value of c.
   b. when a = 3 and b = 4, find the value of c.
4. Pythagoras' Theorem states that $a^2 + b^2 = c^2$ for the sides a, b and c of a right – angled triangle. When a = 5, b = 12 then c = 13.

   Find three more sets of rational numbers for a, b and c which satisfy Pythagoras' Theorem.

Theorem 7.3 (Pythagoras' Theorem)

If a right angled triangle has legs of lengths a and b and hypotenuse of length c, then $a^2+b^2=c^2$.

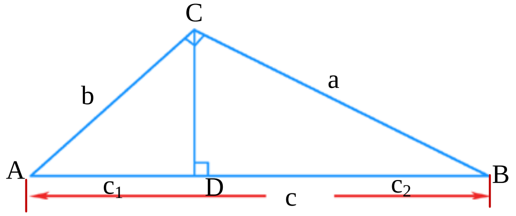


Figure 7.18

Let Δ ABC be right angled triangled the right angle at C as shown above:

Given: Δ ACB is a right triangle and $\overline{CD} \perp \overline{AB}$.

We want to show that : $a^2+b^2 = c^2$.

Proof:

Statements	Reasons
1. $a^2= c_2 \times c$	1. Euclid's Theorem
2. $b^2= c_1 \times c$	2. Euclid's Theorem
3. $a^2+b^2= (c_2 \times c) + (c_1 \times c)$	3. Adding step 1 and 2
4. $a^2+b^2=c(c_1+c_2)$	4. Taking c as a common factor
5. $a^2+b^2=c(c)$	6. Since $c_1+c_2=c$
6. $a^2+b^2=c^2$	5. Proved

Example 4: If a right angle triangle ABC has legs of lengths a= 3cm and b=4cm. What is the length of its hypotenuse?

Solution: Let c be the length of the hypotenuse

$a^2+b^2=c^2$ . . . . Pythagoras' Theorem

$(3cm)^2+(4cm)^2 = c^2$

$9cm^2+16cm^2 = c^2$

$25cm^2 = c^2$

$c = \sqrt{25cm^2}$

$c = 5cm$

Therefore, the hypotenuse is 5 cm long.

Example 5: If a right angle triangle ABC has leg of length a=24cm and the hypotenuse c=25cm. Find the required leg.

Solution: If b is the length of the required leg, then

$a^2+b^2 = c^2$ .... Pythagoras' Theorem

$(24cm)^2+b^2 = (25cm)^2$

$576cm^2+b^2 = 625cm^2$

$b^2 = (625-576)cm^2$

$b^2 = 49cm^2$

$b = \sqrt{49cm^2}$

$b = 7cm$

Therefore, the other leg is 7cm long.

The converse of the Pythagoras' Theorem is stated as follows:

Theorem 7.4 (Converse of Pythagoras' Theorem)

If the lengths of the sides of $\triangle ABC$ are a, b and c where $a^2+b^2=c^2$ then the triangle is right angled. The right angle is opposite the side of length c.

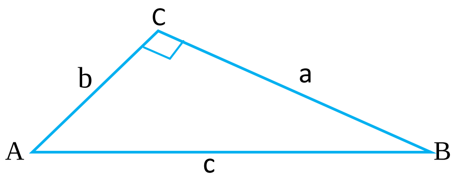


Figure 7.19

The Pythagoras' Theorem and its converse can be summarized as follows respectively.

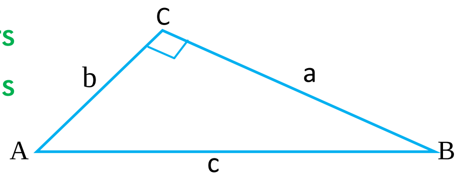


Figure 7.20

In $\triangle ABC$ with a, b lengths of the shorter sides and c the Length of the longest side, then $\triangle$ ABC is right angled if and only if $a^2+b^2=c^2$. Using the Figure above: $\triangle$ ABC is right angled, if and only if $a^2+b^2=c^2$.

Example 6: In Figure 7.21 below. Is ΔABC is a right-angled?

Solution:

i.. $a^2 = (12cm)^2 = 144cm^2$

ii. $b^2 = (5cm)^2 = 25cm^2$

ii. $c^2 = (13cm)^2 = 169cm^2$

Therefore, $a^2 + b^2 = 169cm^2$ and

$c^2 = 169cm^2$

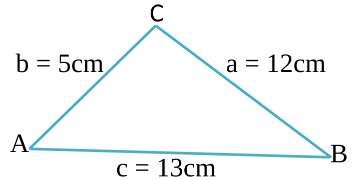


Figure 7.21

Hence ΔABC is right angled, the right angle at C….. converse of Pythagoras theorem.

Example 7: In Figure 7.22 below. Is ΔABC is a right-angled?

Solution:

i) $a^2 = (8cm)^2 = 64cm^2$

ii) $b^2 = (12cm)^2 = 144cm^2$

iii) $c^2 = (15cm)^2 = 225cm^2$

Therefore, $a^2 + b^2 = (64+144)\ cm^2$

$208cm^2$ and $c^2 = 225\ cm^2$

Therefore, $208cm^2 \neq 225cm^2$

Therefore, Δ ABC is not a right-angled.

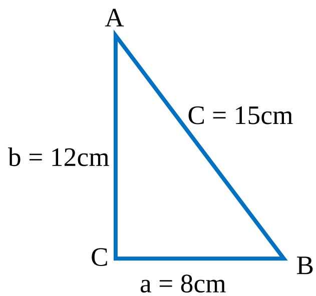


Figure 7.22

Exercise 7B

1. In each of the following Figures ΔABC is a right angled at C. Find the unknown lengths of sides.

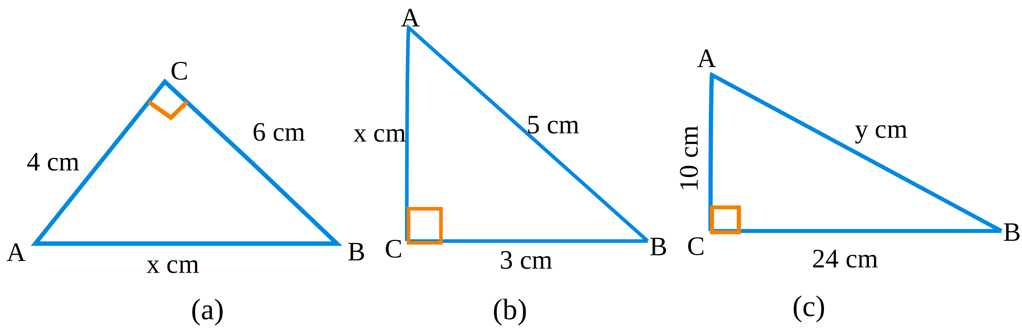


Figure. 7. 23

2. In Figure 7.24 to the right, ABCD is a rectangle with length and width 6 cm and 4 cm respectively. What is the length of the diagonal AC?

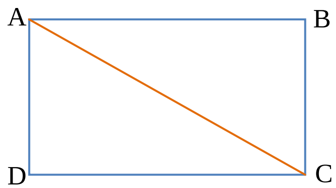


Figure 7.24

3. Find the height of an isosceles triangle with two congruent sides of length 37cm and the base of length 24cm.
4. Abebe and Almaz run 8km east and then 5km north. How far were they from their starting point?
5. A mother Zebra leaves the rest of the herd to go in search of water. She travels due south for 0.9km and, then due east for 1.2km. How far is she from the rest of the herd?

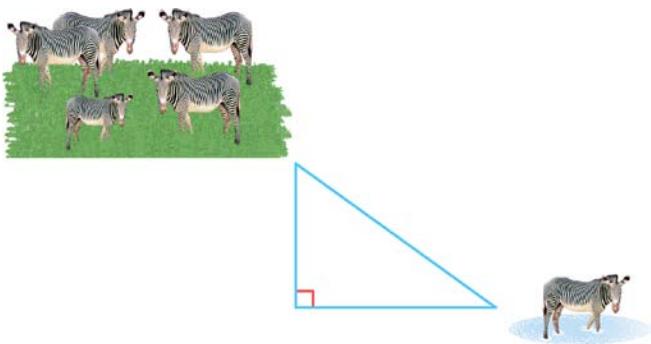


Figure 7.25

6. In Figure 7.26 below ΔABC is an equilateral triangle. $\overline{AD} \perp \overline{BC}$ and $AB = 20$ cm.

   Find: a. AD b. BD c. DC

   Hint: $\overline{AD}$ bisects both $\angle BAC$ and $\overline{BC}$.

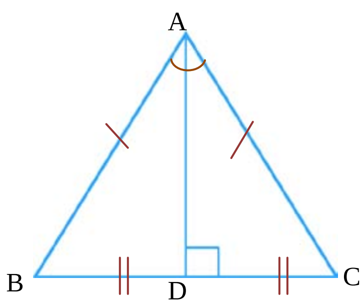


Figure 7.26

7. In Figure 7.27 to the right ABCD is a square and $\overline{DB}$ the diagonal of the square BD= $6\sqrt{2}$ cm. Find the length of side of the square.

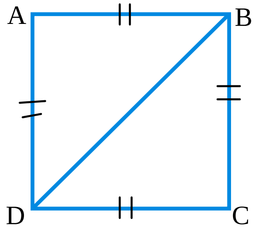


Figure 2.27

8. In Figure 7.28 to the right, Find the length of:
   a. $\overline{BE}$
   b. $\overline{DF}$
   c. $\overline{EF}$
   d. Is ΔCEF is a right-angled?
   e. Is ΔADF is a right-angled?

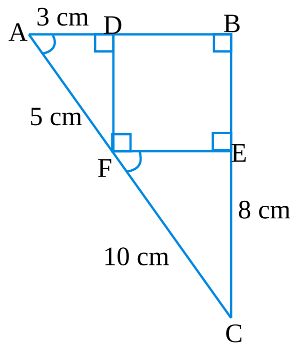


Figure 2.28

9. The right-angled triangle ABC has sides 3cm, 4cm and 5cm. Squares have been drawn on each of its sides.
   a. Find the number of cm squares in:
      i. the square CBFG
      ii. the square ACHI
      iii. the square BADE
   b. Add your answers for (a) (i) and (a) (ii) above.

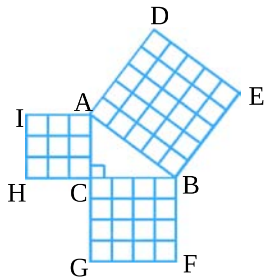


Figure 7.29

   c. State whether or not a triangle with sides of the given lengths is a right triangle.

Using the theorems for calculations

Solving problems using the Euclid's and Pythagoras' Theorems. You are now well aware of the two theorems, their converses and their applications in determining whether a given triangles is right angled triangle. You can now summarize, the Euclid's and the Pythagoras' theorems as follows:

Given: a right angled triangle ABC as shown in the figure to the right and $\overline{CD}$ is altitude to the hypotenuse. Let a, b and c be the side opposite to the angles A, B and C respectively. If AD = c_1 and DB=c_2, then

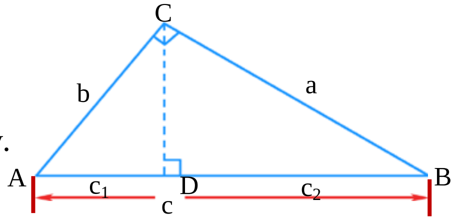


Figure 7.30

$$\left.\begin{array}{l} \text{a. } a^2 = c \times c_2 \\ \quad b^2 = c \times c_1 \end{array}\right\} \text{..........Euclid's Theorem}$$

b. $a^2+b^2 = c^2$ .... Pythagoras' Theorem

Example 8: In Figure 7.31 to the right, if DB = 8cm and AD = 4 cm then find the lengths of:

a. $\overline{AB}$ b. $\overline{BC}$ c. $\overline{AC}$ d. $\overline{DC}$

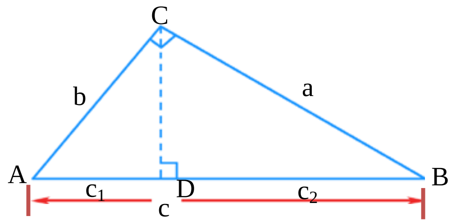


Figure 7.31

Solution:

a. AB = AD + DB . . . . Definition of line segment.

AB = 4cm + 8cm

AB = 12cm

Hence c = 12cm

b. $(BC)^2 = (BD) \times (BA)$ .... Euclid's Theorem

$(BC)^2 = (8cm) \times (12cm)$

$(BC)^2 = 96cm^2$

$BC = 4\sqrt{6}$ cm

c. $(AC)^2=(AD) \times (AB)$ .... Euclid's Theorem

$(AC)^2=(4cm) \times (12cm)$

$(AC)^2 = 48cm^2$

$AC = \sqrt{48cm^2}$

$AC = 4\sqrt{3}cm$

d. $(DC)^2 + (BD)^2 = (BC)^2$..................... Pythagorans' Theorem

$(DC)^2 + (8cm)^2 = (4\sqrt{6}cm)^2$

$(DC)^2 + 64cm^2 = 96cm^2$

$(DC)^2 = (96-64)\ cm^2$

$(DC)^2 = 32\ cm^2$

$DC = \sqrt{32cm^2}$

$DC = 4\sqrt{2}cm$

Example 9: In Figure 7.32 below, find the unknown (marked) length.

$(AC)^2 = (CD) \times (CB)$ .... Euclid's Theorem

$x^2 = 12 \times 16$

$x^2 = 192$ unit square

$x = \sqrt{192}$

$x = 8\sqrt{3}$ unit

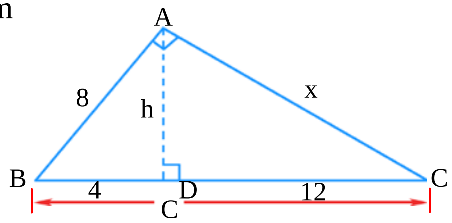


Figure 7.32

Therefore, the value of $x = 8\sqrt{3}$ unit.

$(AD)^2 + (DC)^2 = (AC)^2$ .... Pythagoras' Theorem

$h^2 + (12)^2 = \left(8\sqrt{3}\right)^2$

$h^2 + 144 = 192$

$h^2 = 192 - 144$

$h^2 = 48$

$h = \sqrt{48}$

$h = 4\sqrt{3}$ unit.

Exercise 7C

1. In Figure 7.33, find x, a and b.

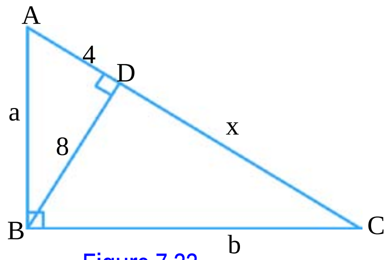


Figure 7.33

2. If p and q are positive integers such that $p > q$. Prove that p^2-q^2, $2pq$ and p^2+q^2 can be taken as the lengths of the sides of a right-angled triangled.

3. How long is an altitude of an equilateral triangle if a side of the triangle is:
   a. 6cm long? b. a cm long?
4. In Figure 7.34 to the right, find x, y and h.

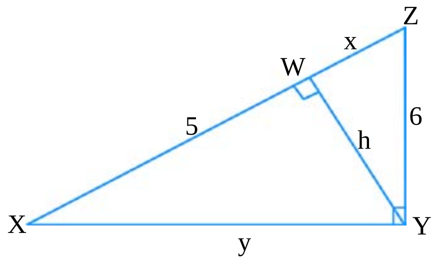


Figure 7.34

7.2 Introduction to Trigonometry

7.2.1 The Trigonometric Ratios

Activity 7.1

Discuss with your teacher

1. **In Figure 7.35 below given a right angled triangle ABC**
   a. **What is**
      i. **the opposite side to angle α?**
      ii. **the adjacent side to angle α?**
      iii. **the hypotenuse of Δ ABC?**
   b. **What is**
      i. **the opposite side to β?**
      ii. **the adjacent side to β?**
      iii. **the hypotenuse of Δ ABC?**

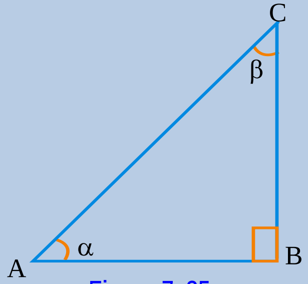


Figure 7. 35

2. **In Figure 7.35 given a right angled triangle ABC:**
   a. **In terms of the lengths AB, BC, AC, write sin α and sin β.**
   b. **In terms of the lengths AB, BC, AC, write cos α and cos β.**

The word trigonometric is derived from two Greek words trigono meaning a triangle and metron meaning measurement. Then the word trigonometry literally means the branch of mathematics which deals with the measurement of triangles. The sine, the cosine and tangent are some of the trigonometric functions.

In this sub unit you are mainly dealing with trigonometric ratios. These are the ratios of two sides of a right angled triangle.

? **What are trigonometric ratios?**

Before defining them let us consider the following Figure 7.36

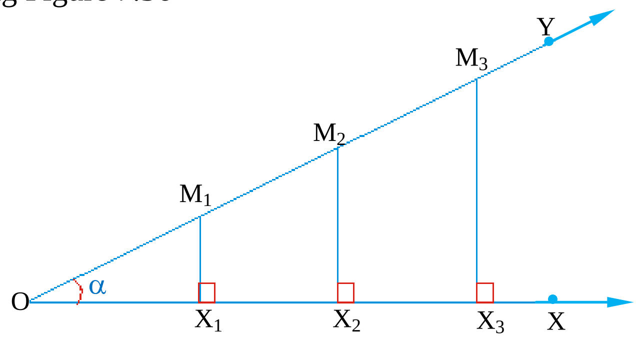


Figure 7. 36

In Figure 7.36 $\overrightarrow{OY}$ and $\overrightarrow{OX}$ are rays that make an acute angle $\overline{X_1M_1}$, $\overline{X_2M_2}$ and $\overline{X_3M_3}$ are any three segments each from $\overrightarrow{OY}$ perpendicular to $\overrightarrow{OX}$. It is obvious to show that $\Delta OX_1M_1 \sim \Delta OX_2M_2 \sim \Delta OX_3M_3$ (by AA Similarity Theorem). Then

i. $\frac{X_1M_1}{OM_1} = \frac{X_2M_2}{OM_2} = \frac{X_3M_3}{OM_3}$ this ratio is called the **sine** of $\angle XOY$ which is abbreviated as:

$$\textbf{Sin}\ (\angle XOY) = \frac{X_1M_1}{OM_1} = \frac{X_2M_2}{OM_2} = \frac{X_3M_3}{OM_3} = \sin\alpha,\ (\textbf{sine} \cong \textbf{sin}).$$

ii. $\frac{OX_1}{OM_1} = \frac{OX_2}{OM_2} = \frac{OX_3}{OM_3}$ this ratio is called the **cosine** of $\angle XOY$ which is abbreviated as:

$$\textbf{Cos}\ (\angle XOY) = \frac{OX_1}{OM_1} = \frac{OX_2}{OM_2} = \frac{OX_3}{OM_3} = \cos\alpha,\ (\text{cosine} \cong \cos).$$

iii. $\frac{X_1M_1}{OX_1} = \frac{X_2M_2}{OX_2} = \frac{X_3M_3}{OX_3}$ this ratio is called the **tangent** of ($\angle XOY$) which is abbreviated as:

$$\textbf{tan}\ (\angle XOY) = \frac{X_1M_1}{OX_1} = \frac{X_2M_2}{OX_2} = \frac{X_3M_3}{OX_3} = \tan\alpha,\ (\text{tangent} \cong \tan).$$

Note: The sine, cosine and tangent are trigonometric ratio depends on the measure of the angle but not on the size of the triangle.

In Figure 7.37 to the right, in a right-triangle ABC, if $\angle C$ is the right-angle, then $\overline{AB}$ is the hypotenuse, $\overline{BC}$ is the side opposite to $\angle A$ and $\overline{AC}$ is the side adjacent to $\angle A$.

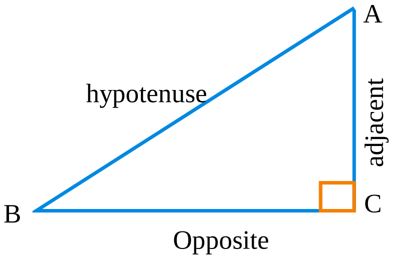


Figure 7.37

Definition 7.1: If Δ ABC is right-angled at C, then

a. sine $\angle A = \dfrac{\text{length of the side opposite to } \angle A}{\text{length of hypotenuse}} = \dfrac{BC}{AB}$

b. Cosine $\angle A = \dfrac{\text{length of the side adjacent to} < A}{\text{length of hypotenuse}} = \dfrac{AC}{AB}$

c. Tangent $\angle A = \dfrac{\text{length of the side opposite to } \angle A}{\text{length of the side adjacent to } \angle A} = \dfrac{BC}{AC}$

Note: i. Sine of $\angle$ A, cosine of $\angle$ A and Tangent of $\angle$ A are respectively abbreviated as sin $\angle$A, cos $\angle$A and tan $\angle$A.

ii. The lengths of the opposite side, adjacent side and hypotenuse are denoted by the abbreviations "opp.", "adj." and "hyp." Respectively.

Example 10: Use Figure 7.38 to state the value of each ratio.

a. sin A
b. cos A
c. tan A

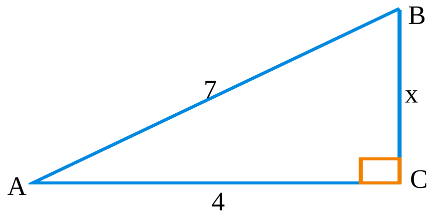


Figure 7.38

Solution:

$(AC)^2 + (BC)^2 = (AB)^2$ .... Pythagoras's Theorem

$4^2 + x^2 = 7^2$

$x^2 = 49 - 16$

$x^2 = 33$

$x = \sqrt{33}$

a. $\text{Sin } A = \frac{\text{opp.}}{\text{hyp.}} = \frac{\sqrt{33}}{7}$

b. $\text{Cos } A = \frac{\text{adj.}}{\text{hyp.}} = \frac{4}{7}$

c. $\tan A = \frac{\text{opp.}}{\text{adj..}} = \frac{\sqrt{33}}{4}$

Exercise 7D

1. Use Figure 7.39 at the right to state the value of each ratio
   a. $\sin\theta$
   b. $\cos\theta$
   c. $\tan\theta$
   d. $\sin\alpha$
   e. $\cos\alpha$
   f. $\tan\alpha$

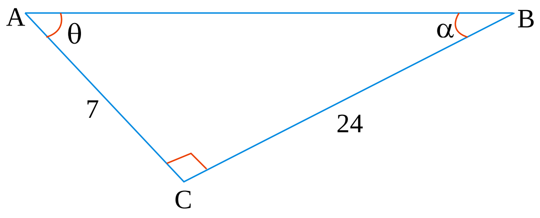


Figure 7.39

2. Use Figure 7.40 at the right to find the value of each ratio.
   a. Find the value of x
   b. $\sin\beta$
   c. $\cos\beta$
   d. $\tan\beta$
   e. $\sin\theta$
   f. $\cos\theta$
   g. $\tan\theta$

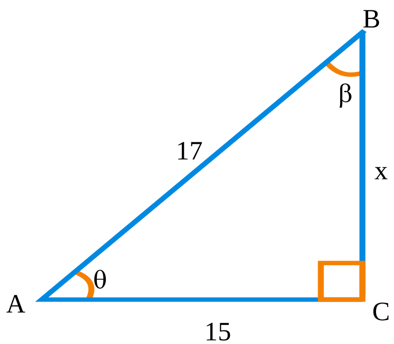


Figure 7. 40

Challenge Problems

3. In Figure 7.41 at the right to state the value of each ratio.

   a. sin β   d. sin α

   b. cos β   e. cos α

   c. tan β   f. tan α

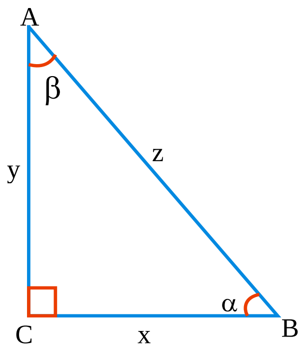


Figure 7.41

4. Use Figure 7.42 at the right to describe each ratio.

   a. $\dfrac{\sin\beta}{\cos\beta}$

   b. $\dfrac{\cos\beta}{\sin\beta}$

   c. $\dfrac{\sin\alpha}{\cos\alpha}$

   d. $\dfrac{\cos\alpha}{\sin\alpha}$

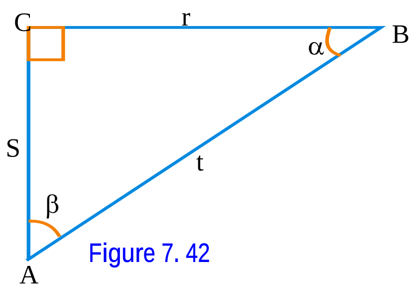


Figure 7. 42

7.2.2 The Values of Sine, Cosine and tangent for 30°, 45° and 60°

The following class activity will help you to find the trigonometric values of the special angle 45°.

Activity 7.2

Discuss with your friends/ parents

Consider the isosceles right angle triangle in Figure 7.43.

a. Calculate the length of the hypotenuse AB.
b. Are the measure angles A and B equal?
c. Which side is opposite to angle A?
d. Which side is adjacent to angle B?
e. What is the measure of angle A?
f. What is the measure of angle B?
g. Find sin ∠A, cos ∠A and tan ∠A.
h. Find sin ∠B, cos B and tan ∠ B.
i. Compare the result (value) of g and h.

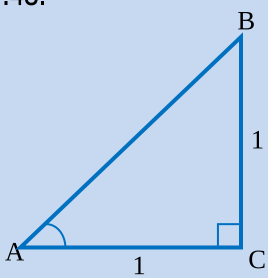


Figure 7.43

From activity 7.3 you have found the values of sin 45^0, cos 45^0 and tan 45^0. In an isosceles right triangle, the two legs are equal in length. Also, the angles opposite the legs are equal in measure. Since

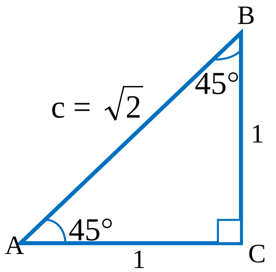


Figure 7.44

$$m(\angle A)+m(\angle B)+m(\angle c) = 180^0 \text{ and}$$
$$m(\angle c) = 90^0$$
$$m(\angle A)+m(\angle B) = 90^0$$

Since $m(\angle A)=m(\angle B)$ each has the measure 45^0.

In Figure 7.44, each legs is 1 unit long. From the Pythagorean property:

$$c^2 = 1^2 + 1^2$$
$$c^2 = 2$$
$$c = \sqrt{2}$$

Hence $\sin 45^0 = \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.... Why?

$\cos 45^0 = \frac{\text{adj.}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.... Why?

$\tan 45^0 = \frac{\text{opp.}}{\text{adj.}} = 1$

Example 11: In Figure 7.45, find the values of x and y.

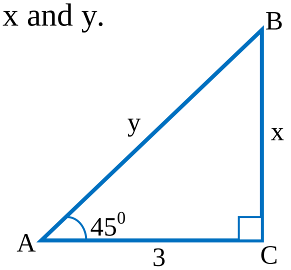


Figure 7.45

Solution:

$$\tan 45° = \frac{\text{opp.}}{\text{adj.}}$$
$$1 = \frac{x}{3}$$
$$x = 3$$
$$\sin 45° = \frac{\text{opp.}}{\text{adj.}}$$
$$\frac{1}{\sqrt{2}} = \frac{3}{y}$$
$$y = 3\sqrt{2}$$

The following Activity will help to find the trigonometric values of the special angles 30° and 60°.

Activity 7.3

Discuss with your friends/ partner

Consider the equilateral triangle ABC with side 2 units long as shown in Figure 7.46 below.

a. Calculate the length of $\overline{AD}$.

b. Calculate the length of $\overline{DC}$.

c. Find sin 30°, cos 30° & tan 30°.

d. Find sin 60°, cos 60° and tan 60°.

e. Compare the results of sin 30° and cos 60°.

f. Compare the results of cos 30° and sin 60°.

g. Compare the results of tan 30° and tan 60°.

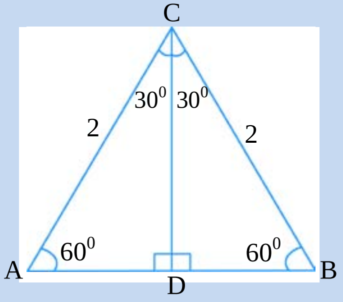


Figure 7.46

From activity 7.3 you have attempted to find the values of $\sin 30^0$, $\cos 30^0$, $\tan 30^0$, $\sin 60^0$, $\cos 60^0$ and $\tan 60^0$. Consider the equilateral triangle in Figure 7.47 with side 2 units. The altitude $\overline{DC}$ bisects $\angle C$ as well as side $\overline{AB}$. Hence $m(\angle ACD)=30^0$ and AD = 1 unit….. (Why)?

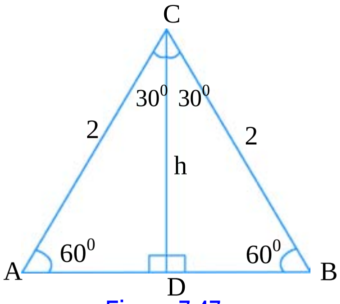


Figure 7.47

$(AD)^2 + (DC)^2 = (AC)^2$ ... Pythagorean Theorem in ΔADC.

$$1^2 + h^2 = 2^2$$
$$h^2 + 1 = 4$$
$$h^2 = 3$$
$$h = \sqrt{3} \text{ units.}$$

Now in the right-angled triangle ADC

Hence, $\sin 30^0 = \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{2}$

$\cos 30^0 = \frac{\text{adj.}}{\text{hyp.}} = \frac{\sqrt{3}}{2}$

$\tan 30^0 = \frac{\text{opp.}}{\text{adj.}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sin 60^0 = \frac{\text{opp.}}{\text{hyp.}} = \frac{\sqrt{3}}{2}$

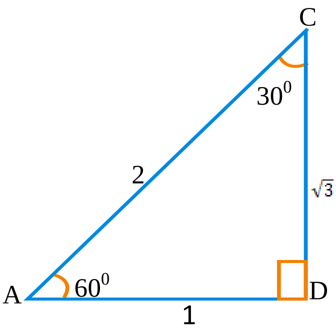


Figure 7.48

$\cos 60^0 = \dfrac{\text{adj.}}{\text{hyp.}} = \dfrac{1}{2}$

$\tan 60^0 = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{\sqrt{3}}{1} = \sqrt{3}$

Example 12: In Figure 7.49, find the values of x and y.

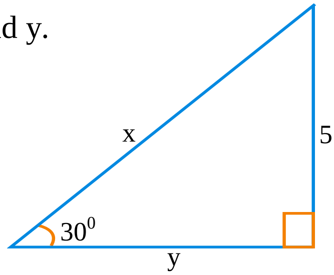


Figure 7.49

Solution:

$\sin 30^0 = \dfrac{\text{opp.}}{\text{hyp.}} = \dfrac{5}{x}$

$\dfrac{1}{2} = \dfrac{5}{x}$

x=10 units

$\tan 30^0 = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{5}{y}$

$\dfrac{\sqrt{3}}{3} = \dfrac{5}{y}$

$\sqrt{3}y = 15$

$y = \dfrac{15}{\sqrt{3}} = 5\sqrt{3}$ units

Example 13: In Figure 7.50, find the values of x and y.

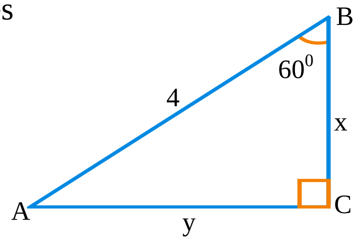


Figure 7.50

Solution:

$\sin 60^0 = \dfrac{\text{opp.}}{\text{hyp.}} = \dfrac{y}{4}$

$\dfrac{\sqrt{3}}{2} = \dfrac{y}{4}$

$2y = 4\sqrt{3}$

$y = 2\sqrt{3}$ unit.

$\cos 60^0 = \dfrac{\text{adj.}}{\text{hyp.}} = \dfrac{x}{4}$

$\dfrac{1}{2} = \dfrac{x}{4}$

$x = 2$ unit.

Example 14: A tree casts a 60 meter shadow and makes an angle of 30^0 with the ground. How tall is the tree?

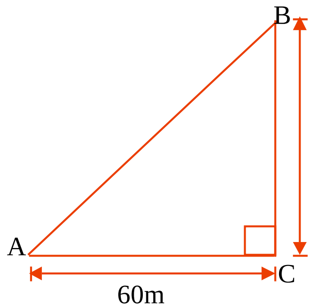


Figure 7.51

Solution: Let Figure 7.51 represent the given problems.

$$\tan \angle A = \frac{\text{opp.}}{\text{adj.}}$$

$$\tan 30^0 = \frac{h}{60\text{ m}}$$

$$h = 60\text{m} \tan 30^0$$

$$h = 60\text{m} \times \frac{\sqrt{3}}{3}$$

$$h = 20\sqrt{3}\ \text{meter}$$

Therefore, the height of the tree is $20\sqrt{3}$ meters.

Example 15: The diagonal of a rectangle is 20cm long, and makes an angle of 30^0 with one of the sides. Find the lengths of the sides of the rectangle.

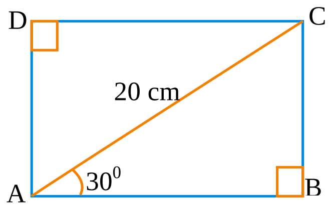


Figure 7.52

Solution: Let Figure 7.52 represent the given problems

$$\sin 30^0 = \frac{\text{opp.}}{\text{hyp.}}$$

$$\sin 30^0 = \frac{BC}{20}$$

$$BC = 20\sin 30^0$$

$$BC = 20 \times \frac{1}{2}$$

$$BC = 10\text{cm}$$

$$\cos 30^0 = \frac{\text{adj.}}{\text{hyp.}}$$

$$\cos 30^0 = \frac{AB}{20\text{ cm}}$$

$$\frac{\sqrt{3}}{2} = \frac{AB}{20\text{ cm}}$$

$$AB = 10\sqrt{3}\text{cm}$$

Therefore, the lengths of the sides of the rectangles are 10cm and 10 $\sqrt{3}$ cm.

Example 16: When the angle of elevation of the sun is 45^0, a building casts a shadow 30m long. How high is the building?

Solution: Let Figure 7.53 represent the given problem

$$\tan 45^0 = \frac{\text{opp.}}{\text{adj.}}$$

$$1 = \frac{h}{30}$$

$$h = 30\text{m}$$

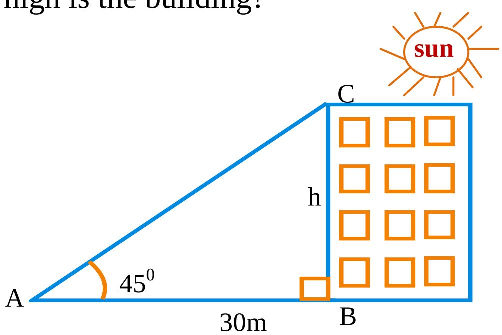


Figure 7.53

Therefore, the height of the Building is 30m.

Example 17: A weather balloon ascends vertically at a rate of 3.86 km/hr while it is moving diagonally at an angle of 60^0 with the ground. At the end of an hour, how fast it moves horizontally (Refer to the Figure 7.54 below).

Solution: Let Figure 7.54 represent the given problem and x be the horizontal speed

$$\tan 60^0 = \frac{\text{opp.}}{\text{adj.}}$$

$$\sqrt{3} = \frac{3.86}{x}\text{km/hr}$$

$$\sqrt{3}x = 3.86\text{ km/hr}$$

$$x = \frac{3.86}{\sqrt{3}}\text{km/hr}$$

$$x = \frac{3.86}{0.5774} = 6.85\text{km/hr}$$

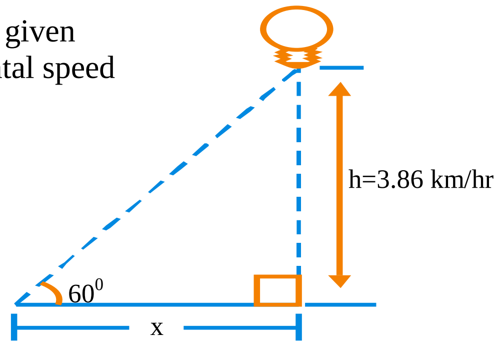


Figure 7.54

Therefore, the horizontal speed of the ballon is 6.85km/hr.

Example 18: A ladder 20 meters long, leans against a wall and makes an angle of 45^0 with the ground. How high up the wall does the ladder reach? And how far from the wall is the foot of the ladder?

Solution: Let in Figure 7.55 represent the given problem

$$\cos 45° = \frac{\text{adj.}}{\text{hyp.}}$$

$$\frac{1}{\sqrt{2}} = \frac{AB}{20\text{ m}}$$

$$20 = \sqrt{2}AB$$

$$AB = \frac{20\text{ m}}{\sqrt{2}} = 10\sqrt{2}\ \text{meters}$$

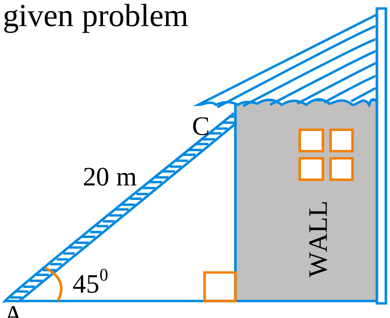


Figure 7.55

Therefore, the foot of the ladder is $10\sqrt{2}$ meters far from the wall.

$$\sin 45^0 = \frac{\text{opp.}}{\text{hyp.}}$$

$$\frac{1}{\sqrt{2}} = \frac{BC}{20\text{ m}}$$

$$20 = \sqrt{2}BC$$

$$BC = \frac{20}{\sqrt{2}} = 10\sqrt{2}\ \text{meters}$$

Therefore, the ladder reaches at $10\sqrt{2}$ meters high far from the ground.

Example 19: At a point A, 30 meters from the foot of a school building as shown in Figure 7.56 to the right, the angle to the top of the building C 60^0. What is the height of the school, building?

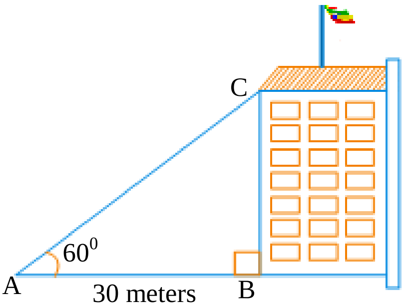


Figure 7.56

Solution:

By considering Δ ABC which is right angled you can use trigonometric ratio.

$$\tan 60^0 = \frac{\text{opp.}}{\text{adj.}}$$

$$\tan 60^0 = \frac{BC}{AB}$$

$$\sqrt{3} = \frac{h}{30\text{m}}$$

$$h = 30\sqrt{3}\text{meters}$$

Therefore, the height of the school building is $30\sqrt{3}$ meters.

Exercise 7E

1. In Figure 7.57 below find the value of x.

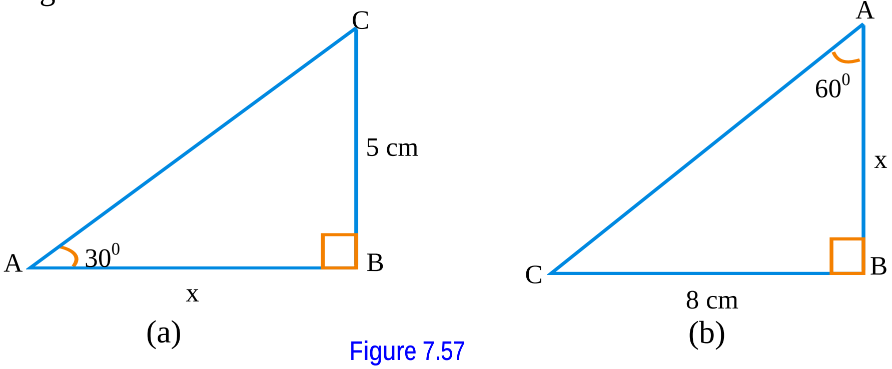


Figure 7.57

2. A ladder of length 4m leans against a vertical wall so that the base of the ladder is 2 meters from the wall. Calculate the angle between the ladder and the wall.
3. A ladder of length 8m rests against a wall so that the angle between the ladder and the wall is 45^0. How far is the base of the ladder from the wall?
4. In Figure 7.58 below, a guide wire is used to support a 50 meters radio antenna so that the angle of the wire makes with the ground 60^0. How far is the wire is anchored from the base of the antenna?

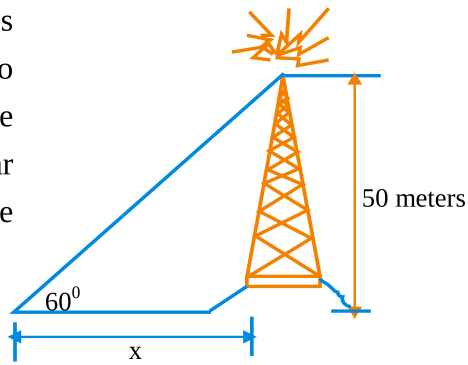


Figure 7.58

5. In an isosceles right triangle the length of a leg is 3cm. How long is the hypotenuse?

Challenge problems

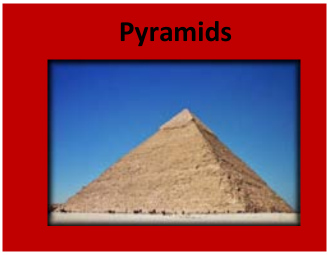
6. How long is an altitude of an equilateral triangles, if the length of a side of the triangle is

a. 6cm b. 4cm c. 10 cm

7. In a 45^0-45^0-90^0 triangle the length of the hypotenuse is 20cm. How long is its leg?

7.3 Solids Figures

7.3.1Pyramid



Historical note

The Egyptian pyramids are ancient pyramid shaped brick work structures located in Egypt. The shape of a pyramid is thought to be representative of the descending rays of the sun.

Group work 7.3

Discuss with your friends/ partners.

1. What is a pyramid?
2. Can you give a model or an example of a pyramid
3. Answer the following question based on the given Figure 7.59 below.
   a. Name the vertex of the pyramid.
   b. Name the base of the pyramid.
   c. Name the lateral faces of the pyramid.
   d. Name the height of the pyramid.
   e. Name the base edge of the pyramid.
   f. Name the lateral edge of the pyramid.

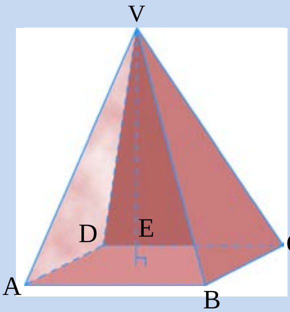


Figure 7.59 Rectangular pyramid

Definition 7.2: *A Pyramid* is a solid figure that is formed by line segments joining every point on the sides and every interior points of a polygonal region with a point out side of the plane of the polygon.

From the group work (7.3) above you may discuss the following terminologies.

- ✓ The polygonal region ABCD is called the **base** of the pyramid.
- ✓ The point **V** outside of the plane of the polygon (base) is called the **vertex** of the pyramid.
- ✓ The triangles VAB, VBC, VCD, and VDA are called **lateral faces** of the pyramid (see Figure 7.59).
- ✓ $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$ are the edges of the base of the pyramid (see Figure 7.59).
- ✓ $\overline{VA}, \overline{VB}, \overline{VC}$ and $\overline{VD}$ are **lateral edge** of the pyramid (see Figure 7.59).
- ✓ The **altitude of a pyramid** is the perpendicular distance from the vertex to the point of the base.
- ✓ The **slant height** is the length of the altitude **of a lateral face** of the pyramid.
- ✓ Generally look at Figure 7.60 below.

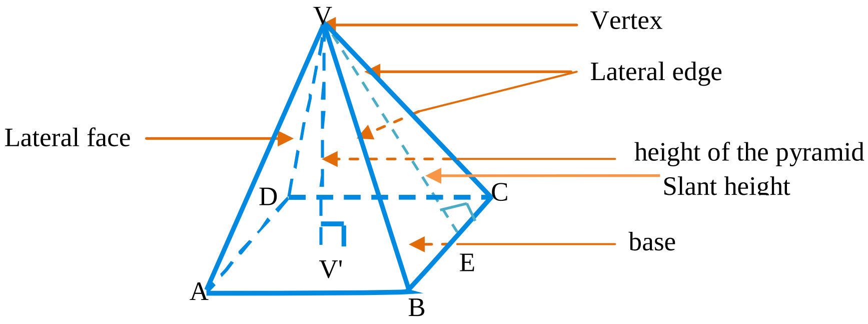
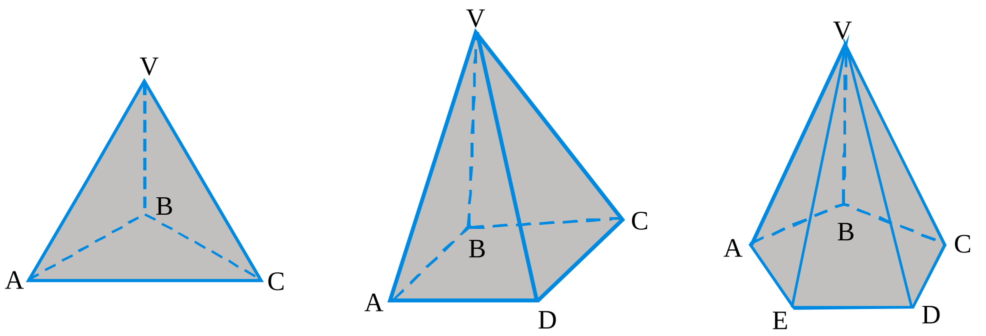


Figure 7.60 Rectangular pyramid

Figure 7.61 below show different pyramids. The shape of the base determines the name of the pyramid.



(a) Triangular pyramid (b) Quadrilateral pyramid (c) Pentagonal pyramid

Figure 7.61

Activity 7.4

Discuss with your teacher before starting the lesson.

1. Make a list of the names of these shapes. You do not have to draw them. Choose from: hexagonal pyramid, tetrahedron, and square pyramid.
2. What is a regular pyramid?
3. What is altitude of the pyramid?

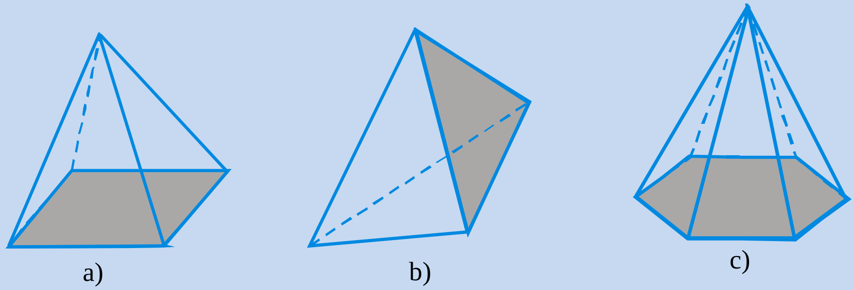


Figure 7.62

Special class of pyramids are known as **right pyramids.** To have a right pyramid the following condition must be satisfied: The foot of the altitude must be at the **center of the base.** In Figure 7.63 to the right shows **a rectangular right pyramid**. The other class of right pyramids are known as **regular pyramids**. To have a regular pyramid, the following three conditions must be fulfilled:

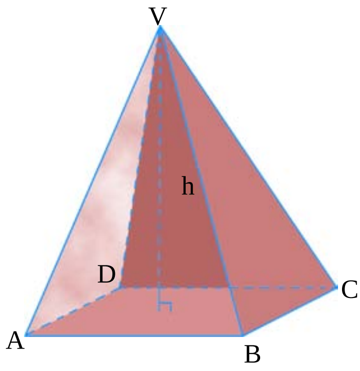


Figure 7.63 Rectangular right pyramid

1. The pyramid must be a right pyramid.
2. The base of the pyramid must be a regular polygon. In Figure 7.64 shows regular pyramids.
3. The lateral edges of a regular pyramid are all equal in length.

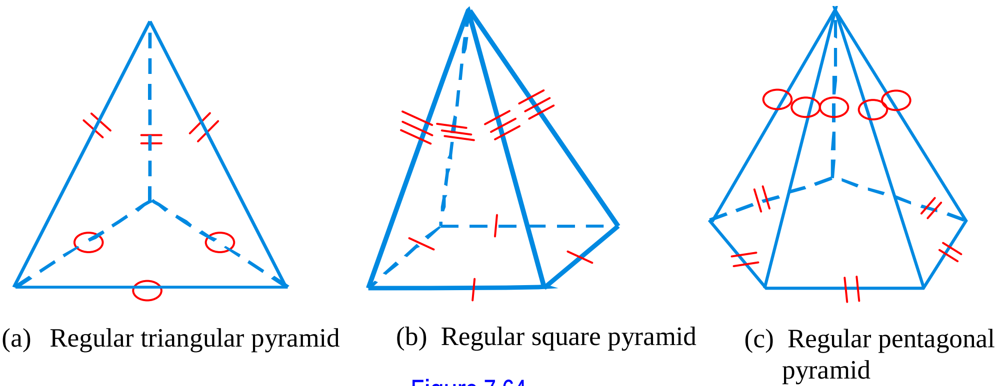


Figure 7.64

Exercise 7F

1. In Figure 7.65 shows a square pyramid.
   a. Name its vertex.
   b. Name its four lateral edges.
   c. Name its four lateral faces.
   d. Name the height.
   e. Name the base.

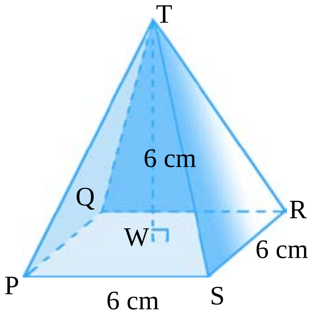


Figure 7.65

2. In Figure 7.66 to the right
   a. Name the vertex of the pyramid
   b. Name the lateral edge of the pyramid
   c. Name the lateral faces of the pyramid.

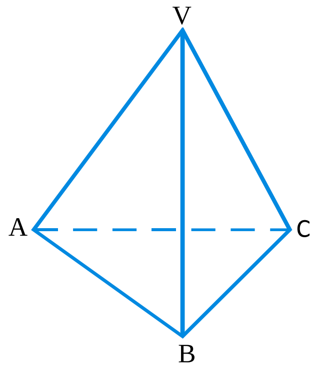


Figure 7.66 Pyramid

7.3.2 Cone

Group Work 7.4

Discuss with your friends.

1. What is a cone?
2. Answer the following question based on the given Figure 7.67 to the right.
   a. Name the vertex of the cone.
   b. Name the slant height of the cone.
   c. Name the base of the cone.
   d. Name the altitude of the cone.

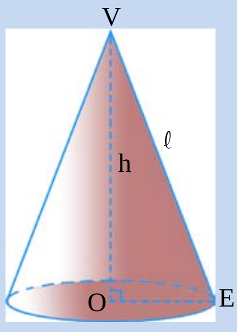


Figure 7.67

Definition7.3: The solid figure formed by joining all points of a circle to a point not on the plane of the circle is called **cone**.

In Figure 7.68, represent a cone.

The original circle is called the **base** of the cone and the curved closed surface is its **lateral surface**.

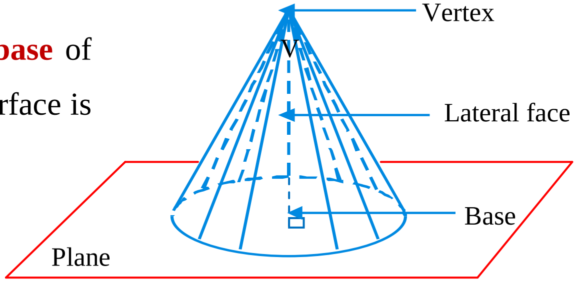


Figure 7.68

The point outside the plane and at which the segments from the circular region joined is called the **vertex** of the cone.

The perpendicular distance from the base to the vertex is called the **altitude** of the cone.

Definition7.4: a. *A Right Circular* cone is a circular cone with the foot of its altitude is at the center of the base as shown in Figure 7.69 to the right.

b. A line segment from the vertex of a right circular cone to any point of the circle is called the slant height.

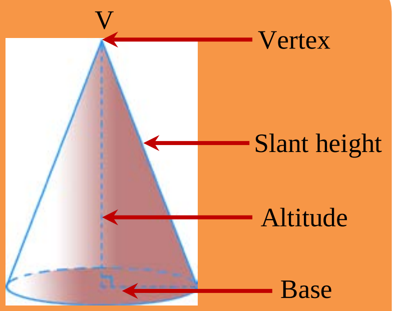


Figure 7.69

Exercise 7G

1. Draw a cone and indicate:

   a. the base
   b. the lateral face
   c. the altitude
   d. the slant height
   e. the vertex

2. What is right circular cone?
3. What is oblique circular cone?

Summary For Unit 7

Given

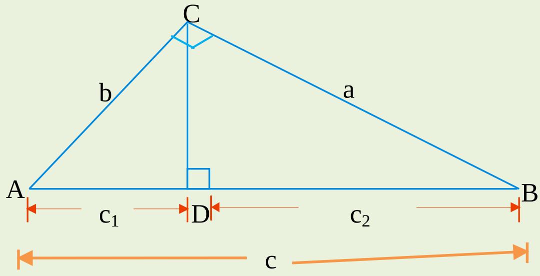


Figure 7.70

For 1-4 below refer to right triangle ABC in Figure 7.70 above.

1. ***Euclid's Theorem*** *i)* $a^2 = c_2 \times c$

   ii) $b^2 = c_1 \times c$

2. ***Converse of Euclid's Theorem.***

   $a^2 = c_2 \times c$ *and* $b^2 = c_1 \times c$ *if and only if* ΔABC *is right angled.*

3. ***Pythagorean Theorem***: $a^2+b^2=c^2$.

4. ***Converse of Pythagorean Theorem***

   If $a^2+b^2=c^2$, *then* ΔABC *is right angled.*

5. *Trigonometric ratio in right triangle ABC where* $\angle C$ *is the right angle (see Figure 7.71).*

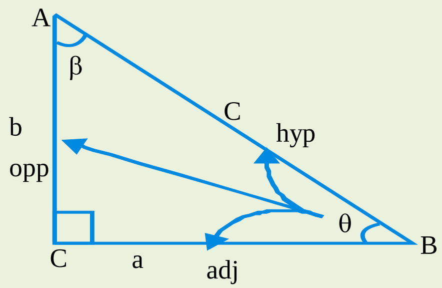


Figure 7.71

✓ $\overline{BC}$ *is the side* ***adjacent (adj)*** *to angle* θ.

✓ *is the side* ***opposite (opp)*** *to angle* θ.

✓ $\overline{AB}$ *is the* ***hypotenuse (hyp)*** *to angle* θ

a. $\sin\theta = \frac{opp.}{hyp.} = \frac{b}{c}$

b. $\cos\theta = \frac{adj.}{hyp.} = \frac{a}{c}$

c. $\tan\theta = \frac{opp.}{adj.} = \frac{b}{a}$

d. $\sin\beta = \frac{opp.}{hyp.} = \frac{a}{c}$

e. $\cos\beta = \frac{adj.}{hyp.} = \frac{b}{c}$

f. $\tan\beta = \frac{opp.}{adj.} = \frac{a}{b}$

6. *Referring to the values given in table 7.2 below.*

θ	Sinθ	cosθ	tanθ
30^0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45^0	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60^0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

7. *Relationship between 30^0 and 60^0 as follows:*

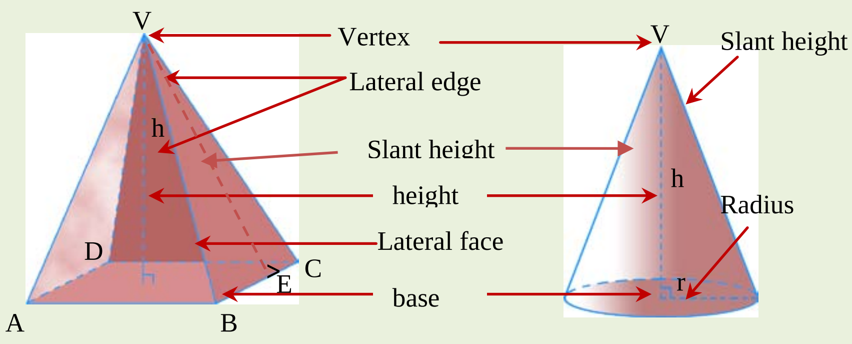
a. $\sin 60^0 = \cos 30^0 = \frac{\sqrt{3}}{2}$

b. $\cos 60^0 = \sin 30 = \frac{1}{2}$

8. ***A Pyramid*** *is a solid figure that is formed by line segments joining every point on the sides and every interior points of a polygonal region with a point out side of the plane of the polygon.*

9. *The solid figure formed by joining all points of a circular region to a point outside of the plane of the circle is called* ***a circular cone****.*

10.



(a) Rectangular pyramid (b) Right circular cone

Figure 7.72

Miscellaneous Exercise 7

I. Choose the correct answer from the given alternatives.

1. A rectangle has its sides 5cm and 12cm long. What is the length of its diagonals?

   a. 17cm   b. 13 cm   c. 7cm   d. 12cm

2. In Figure 7.73 to the right

   m ($\angle$ACB)=90^0 and $\overline{CD} \perp \overline{AB}$.

   If $CD = 10cm$ and $BD = 8cm$, then what is the length of $\overline{AD}$?

   a. $2\sqrt{41}$ cm   c. $\sqrt{41}$ cm

   b. $4\sqrt{41}$ cm   d. $\frac{25}{2}$cm

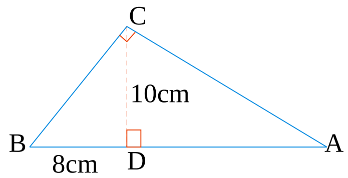


Figure 7.73

3. In Figure 7.74 to the right, right angle triangle XYZ is a right angled at Y and N is the foot of the perpendicular from Y to $\overline{XZ}$. Given that XY= 6cm and XZ=10cm. What is the length of $\overline{XN}$?

   a. 2.4 cm   c. 4.3 cm

   b. 3.6 cm   d. 4.8 cm

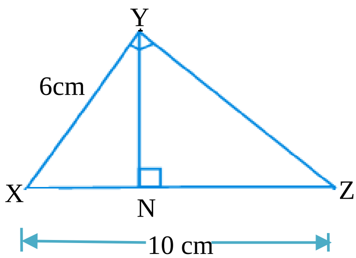


Figure 7.74

4. An electric pole casts a shadow of 24 meters long. If the tip of the shadow is 25 meters far from the top of the pole, how high is the pole from the ground?

   a. 9 meters   c. 7 meters

   b. 10 meters   d. 5 meters

5. Which of the following set of numbers could not be the length of sides of a right angled triangle?

   a. 0.75, 1, 1.25   c. 6, 8, 10

   b. 1, $\frac{3}{2}$, 2   d. 5, 12, 13

6. A tree 18 meters high is broken off 5 meters from the ground. How far from the foot of the tree will the top strike the ground.

   a. 12 meters   b. 13 meters   c. 8 meters   d. 20 meters

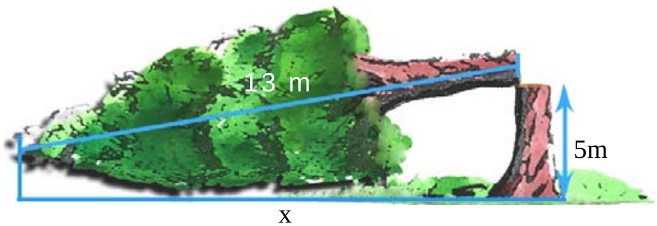


Figure 7.75

7. In Figure 7.76 below ΔABC is right angled at C. if BC=5 and AB=13, then which of the following is true?

   a. $\sin\theta = \frac{12}{13}$

   b. $\tan\beta = \frac{12}{5}$

   c. $\cos\theta = \frac{5}{13}$

   d. $\cos\beta = \frac{13}{5}$

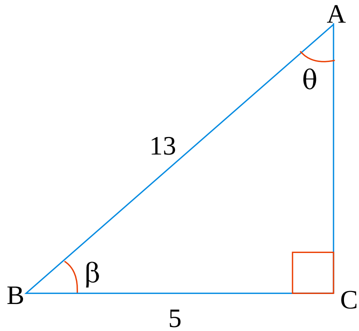


Figure 7.76

8. In Figure 7.77 below, what is the value of x?

   a. 6
   b. 20
   c. 10
   d. 0

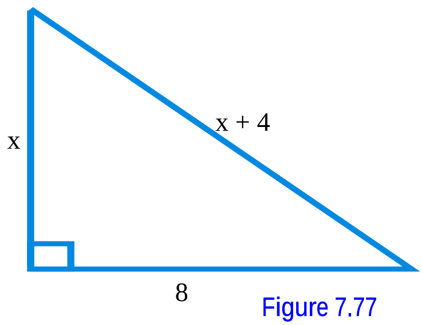


Figure 7.77

9. One leg of an isosceles right triangle is 3cm long. What is the length of the hypotenuse?

   a. 3cm   b. $3\sqrt{2}$ cm   c. $3\sqrt{3}$ cm   d. $\sqrt{6}$ cm

10. In Figure 7.78 below which of the following is true?

a. $\sin \angle A = \frac{16}{22}$

b. $\cos \angle B = \frac{16}{22}$

c. $\tan \angle B = \frac{\sqrt{57}}{8}$

d. All are correct answer

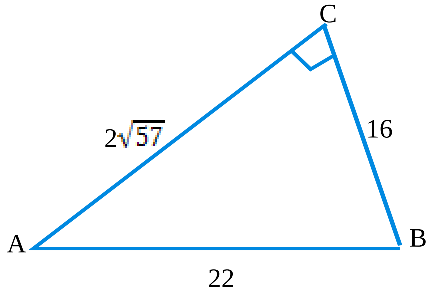


Figure 7.78

11. In Figure 7.79 below, which of the following is true about the value of the variables?

a. $x=2\sqrt{3}$

b. $y=6$

c. $z=4\sqrt{3}$

d. All are true

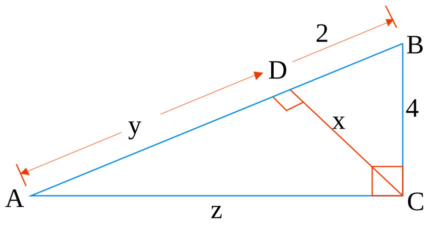


Figure 7.79

12. In Figure 7.80 to the right What is the value of x?

a. 6

b. 18

c. $3\sqrt{2}$

d. $-3\sqrt{2}$

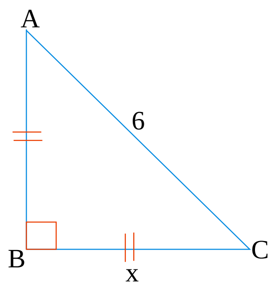


Figure 7.80

13. In Figure 7.81 below, what is the value of x?

a. $10\sqrt{3}$

b. $\frac{30}{\sqrt{3}}$

c. $\frac{30\sqrt{3}}{3}$

d. All are true

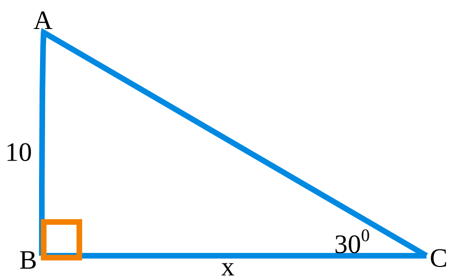


Figure 7.81

14. Which of the following is true about given ΔPQR given in Figure 7.82 to the right?
    a. $p^2+q^2=r^2$
    b. $q^2+r^2=p^2$
    c. $(p+q)^2=r^2$
    d. $p^2+r^2=q^2$

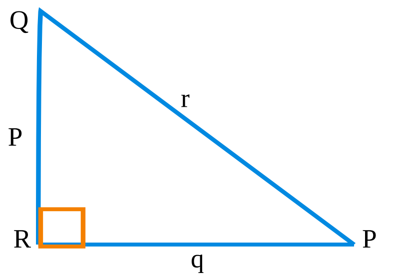


Figure 7.82

15. In Figure 7.83 to the right, find the length of the side of a rhombus whose diagonals are of length 6 and 8 unit.
    a. 14 units
    b. 5 units
    c. 10 unit
    d. 15 unit

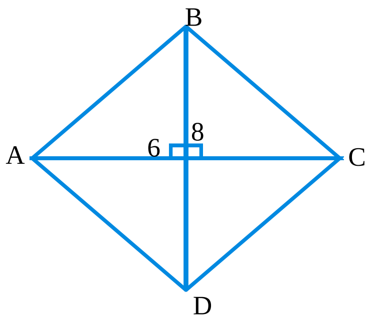


Figure 7.83

II. Work out Question

16. In Figure 7.84 to the right, $\overline{CD} \perp \overline{AB}$ AD = 4 cm, CD = 9cm and DB=14cm. Is ΔABC a right angled?

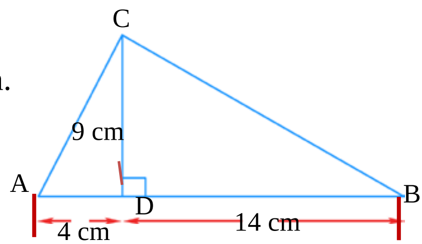


Figure 7.84

17. ΔABC is a right-angled triangle as shown in Figure 7.85 to the right. If AD = 12cm BD = DC then find the lengths of $\overline{BD}$ and $\overline{DC}$.

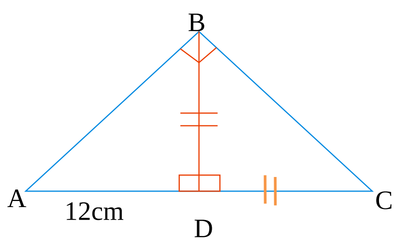


Figure 7.85

18. In Figure 7.86 to the right, find the value of the variables.

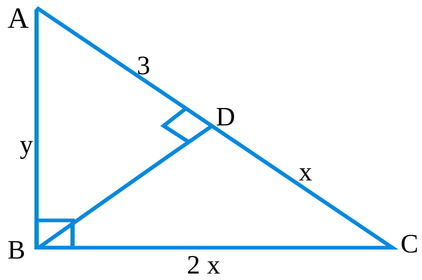


Figure 7.86

19. A triangle has sides of lengths 16, 48 and 50. Is the triangle a right-angled triangle?

20. In Figure 7.87 to the right, if AC = 12 cm, BC = 5 cm, CD = 11 cm, then find

a. $\overline{AD}$ b. $\overline{AB}$

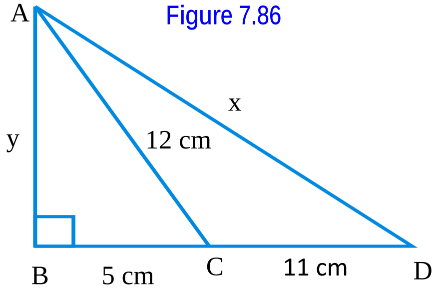


Figure 7.87

21. Let ΔABC be an isosceles triangle and $\overline{AD}$ be its altitude. If the length of side AC = 4x + 4y, BD = 6x, DC = 2x + 2y and AB = 12, then find the length of:

a. $\overline{AC}$
b. $\overline{AD}$
c. $\overline{BC}$
d. $\overline{BD}$
e. $\overline{DC}$

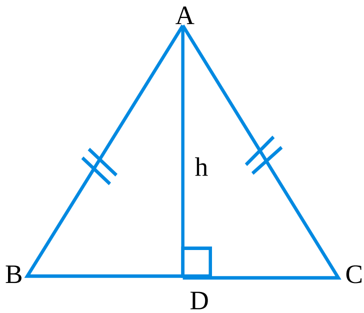


Figure 7.88

22. In Figure 7.89 to the right, what is the value of x, if $\sin B = \frac{2}{3}$.

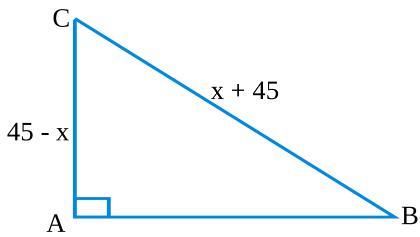


Figure 7.89

23. In Figure 7.90 to the right, what is the value of x, if $\tan\angle D = \frac{8}{5}$.

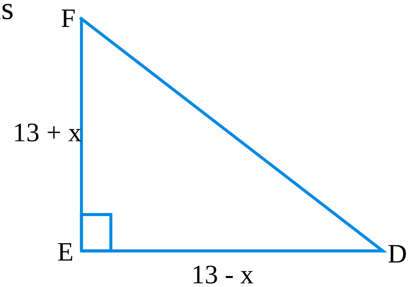


Figure 7.90

24. In Figure 7.91 to the right, what is the value of x, if $\cos c = \frac{2}{5}$.

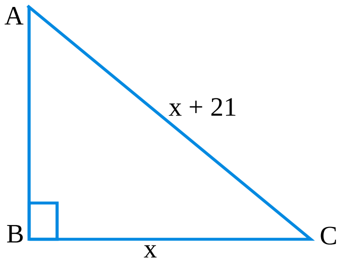


Figure 7.91

25. In Figure 7.92 below, if BC = 5 and AB = 13, then find

a. $\sin\alpha$

b. $\cos\alpha$

c. $\tan\alpha$

d. $\sin\beta$

e. $\cos\beta$

f. $\tan\beta$

g. $\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}$

h. $(\sin\alpha)^2+(\cos\alpha)^2$

i. $(\cos\beta)^2+(\sin\beta)^2$

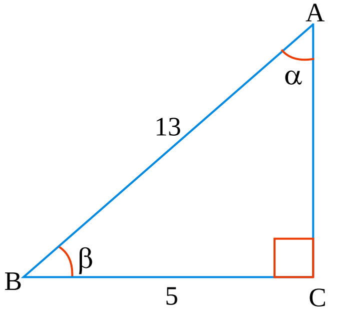


Figure 7.92

The Function $y = x^2$ $\quad 1.00 \le x \le 5.99$

X	0	1	2	3	4	5	6	7	8	9
1.0	**1.000**	**1.020**	**1.040**	**1.061**	**1.082**	**1.102**	**1.124**	**1.145**	**1.166**	**1.188**
1.1	**1.210**	**1.232**	**1.254**	**1.277**	**1.277**	**1.322**	**1.346**	**1.369**	**1.392**	**1.416**
1.2	**1.440**	**1.464**	**1.488**	**1.513**	**1.513**	**1.562**	**1.588**	**1.613**	**1.638**	**1.644**
1.3	**1.690**	**1.716**	**1.742**	**1.769**	**1.769**	**1.822**	**1850**	**1.877**	**1.904**	**1.932**
1.4	**1.960**	**1.988**	**2.016**	**2.045**	**2.045**	**2.102**	**2.132**	**2.161**	**2.190**	**2.220**
1.5	**2.250**	**2.280**	**2316**	**2.341**	**2.341**	**2.102**	**2.434**	**2.465**	**2.496**	**2.528**
1.6	**2.560**	**2.592**	**2.310**	**2.657**	**2.657**	**2.722**	**2.756**	**2.789**	**2.822**	**2.856**
1.7	**2.890**	**2.924**	**2.624**	**2.993**	**2.993**	**3.062**	**3.098**	**3.133**	**3.168**	**3.204**
1.8	**3.240**	**3.276**	**2.958**	**3.349**	**3.349**	**3.422**	**3.460**	**3.497**	**3.534**	**3.572**
1.9	**3.610**	**3.648**	**3.686**	**3.725**	**3.764**	**3.802**	**3.842**	**3.881**	**3.920**	**3.960**
2.0	**4.000**	**4.040**	**4.080**	**4.121**	**4.162**	**4.202**	**4.244**	**4.285**	**4.326**	**4.368**
2.1	**4.410**	**4.452**	**4.494**	**4.537**	**4.580**	**4.622**	**4.666**	**4.709**	**4.752**	**4.796**
2.2	**4.840**	**4.884**	**4.928**	**4.973**	**5.018**	**5.062**	**5.108**	**5.153**	**5.198**	**5.244**
2.3	**5.290**	**5.336**	**5.382**	**5.429**	**5.476**	**5.522**	**5.570**	**5.617**	**5.664**	**5.712**
2.4	**5.760**	**5.808**	**5.856**	**5.905**	**5.954**	**6.002**	**6.052**	**6.101**	**6.150**	**6.200**
2.5	**6.250**	**6.300**	**6.350**	**6.401**	**6.452**	**6.502**	**6.554**	**6.605**	**6.656**	**6.708**
2.6	**6.760**	**6.812**	**6.864**	**6.917**	**6.970**	**7.022**	**7.076**	**7.129**	**7.182**	**7.236**
2.7	**7.290**	**7.344**	**7.398**	**7.453**	**7.508**	**7.562**	**7.618**	**7.673**	**7.728**	**7.784**
2.8	**7.840**	**7.896**	**7.952**	**8.009**	**8.066**	**8.122**	**8.180**	**8.237**	**8.294**	**8.352**
2.9	**8.410**	**8.468**	**8.526**	**8.585**	**8.644**	**8.702**	**8.762**	**8.821**	**8.880**	**8.940**
3.0	**9.000**	**9.060**	**9.120**	**9.181**	**9.242**	**9.302**	**9.364**	**9.425**	**9.486**	**9.548**
3.1	**9.610**	**9.672**	**9.734**	**9.797**	**9.860**	**9.922**	**9.986**	**10.05**	**10.11**	**10.18**
3.2	**10.24**	**10.30**	**10.37**	**10.43**	**10.50**	**10.56**	**10.63**	**10.69**	**10.76**	**11.82**
3.3	**10.89**	**10.96**	**11.02**	**11.09**	**11.16**	**11.22**	**11.29**	**11.36**	**11.42**	**11.49**
3.4	**11.56**	**11.63**	**11.70**	**11.76**	**11.83**	**11.90**	**11.97**	**12.04**	**12.11**	**12.18**
3.5	**14.25**	**12.32**	**12.39**	**12.46**	**12.53**	**12.60**	**12.67**	**12.74**	**12.82**	**12.89**
3.6	**12.96**	**13.03**	**13.10**	**13.18**	**13.25**	**13.32**	**13.40**	**13.47**	**13.54**	**13.62**
3.7	**13.69**	**13.76**	**13.84**	**13.91**	**13.99**	**14.06**	**14.14**	**14.21**	**14.29**	**14.36**
3.8	**14.44**	**14.52**	**14.59**	**14.67**	**14.75**	**14.82**	**14.90**	**14.98**	**15.08**	**15.13**
3.9	**15.21**	**15.29**	**15.37**	**15.44**	**15.52**	**15.60**	**15.68**	**15.76**	**15.84**	**15.92**
4.0	**16.00**	**16.08**	**16.16**	**16.24**	**16.32**	**16.40**	**16.48**	**16.56**	**16.65**	**16.73**
4.1	**16.81**	**16.89**	**16.97**	**17.06**	**17.14**	**17.22**	**17.31**	**17.39**	**17.47**	**17.56**
4.2	**17.64**	**17.72**	**17.81**	**17.89**	**17.98**	**18.06**	**18.15**	**18.23**	**18.32**	**18.40**
4.3	**18.49**	**18.58**	**18.66**	**18.75**	**18.84**	**18.92**	**19.01**	**19.10**	**19.18**	**19.27**
4.4	**19.96**	**19.45**	**19.54**	**19.62**	**19.71**	**19.80**	**19.89**	**19.98**	**20.98**	**20.16**
4.5	**20.25**	**20.34**	**20.43**	**20.52**	**20.61**	**20.70**	**20.79**	**20.88**	**21.90**	**21.07**
4.6	**21.16**	**21.25**	**21.34**	**21.44**	**21.53**	**21.62**	**21.72**	**21.81**	**22.85**	**22.00**
4.7	**22.09**	**22.18**	**22.28**	**22.37**	**22.47**	**22.56**	**22.66**	**22.75**	**23.81**	**22.94**
4.8	**23.04**	**23.14**	**23.33**	**23.33**	**23.43**	**23.52**	**23.62**	**23.72**	**24.80**	**23.91**
4.9	**24.01**	**24.11**	**24.24**	**24.30**	**24.40**	**24.50**	**24.60**	**24.70**	**25.81**	**24.90**
5.0	**25.00**	**25.10**	**25.20**	**25.30**	**25.40**	**25.50**	**25.60**	**25.70**	**26.83**	**25.91**
5.1	**26.01**	**26.11**	**26.21**	**26.32**	**26.42**	**26.52**	**26.63**	**26.73**	**27.88**	**26.94**
5.2	**27.04**	**27.14**	**27.25**	**27.35**	**27.46**	**27.56**	**27.67**	**27.77**	**28.94**	**27.98**
5.3	**28.09**	**28.20**	**28.30**	**28.41**	**28.52**	**28.62**	**28.73**	**28.84**	**28.94**	**29.05**
5.4	**29.16**	**29.27**	**29.38**	**29.48**	**29.59**	**29.70**	**29.81**	**29.92**	**30.03**	**30.14**
5.5	**30.25**	**30.36**	**30.47**	**30.58**	**30.69**	**30.80**	**30.91**	**31.02**	**31.14**	**31.25**
5.6	**31.36**	**31.47**	**31.58**	**31.70**	**31.81**	**31.92**	**32.04**	**32.15**	**32.26**	**32.38**
5.7	**32.46**	**32.60**	**32.72**	**32.83**	**32.95**	**33.06**	**33.18**	**33.29**	**33.41**	**33.52**
5.8	**33.64**	**33.76**	**33.87**	**33.99**	**34.11**	**34.22**	**34.34**	**34.46**	**34.57**	**34.69**
5.9	**34.81**	**34.93**	**35.05**	**35.16**	**35.28**	**35.40**	**35.52**	**35.64**	**35.76**	**35.88**

If you move the comma in x one digit to the right (left), then the comma in x^2 must be moved two digits to the right (left)

The Function $y = x^2$ $6.00 \le x \le 9.99$

X	0	1	2	3	4	5	6	7	8	9
6.0	**36.00**	**36.12**	**36.24**	**36.36**	**36.48**	**36.60**	**36.72**	**36.84**	**36.97**	**37.09**
6.1	**37.21**	**37.33**	**37.45**	**37.58**	**37.70**	**37.82**	**37.95**	**38.07**	**38.19**	**38.32**
6.2	**38.44**	**38.56**	**38.69**	**38.81**	**38.94**	**39.06**	**39.19**	**39.31**	**39.44**	**39.56**
6.3	**39.69**	**39.82**	**39.94**	**40.07**	**40.20**	**40.32**	**40.45**	**40.58**	**40.70**	**40.83**
6.4	**40.96**	**41.09**	**41.22**	**41.34**	**41.47**	**41.60**	**41.73**	**41.86**	**41.99**	**42.12**
6.5	**42.25**	**42.38**	**42.51**	**42.64**	**42.77**	**42.90**	**43.03**	**43.16**	**43.30**	**43.43**
6.6	**43.56**	**43.69**	**43.82**	**43.96**	**44.09**	**44.22**	**44.36**	**44.49**	**44.62**	**44.47**
6.7	**44.89**	**45.02**	**45.16**	**45.29**	**45.43**	**45.56**	**45.70**	**45.63**	**45.63**	**46.10**
6.8	**46.24**	**46.38**	**46.51**	**46.65**	**46.79**	**46.92**	**47.06**	**47.20**	**47.20**	**47.47**
6.9	**47.61**	**47.75**	**47.89**	**48.02**	**48.16**	**48.30**	**48.44**	**48.58**	**48.58**	**48.86**
7.0	**49.00**	**49.14**	**49.28**	**49.42**	**49.56**	**49.70**	**49.84**	**49.98**	**49.98**	**50.27**
7.1	**50.41**	**50.55**	**50.69**	**50.84**	**50.98**	**51.12**	**51.27**	**51.41**	**54.41**	**51.70**
7.2	**51.84**	**51.98**	**52.13**	**52.27**	**52.42**	**52.56**	**52.71**	**52.85**	**52.85**	**53.14**
7.3	**53.29**	**53.44**	**53.58**	**53.73**	**53.88**	**54.02**	**54.17**	**54.32**	**54.32**	**54.61**
7.4	**54.76**	**54.91**	**55.06**	**55.20**	**55.35**	**55.50**	**55.65**	**55.80**	**55.80**	**56.10**
7.5	**56.25**	**56.40**	**56.55**	**56.70**	**56.85**	**57.00**	**57.15**	**57.30**	**57.30**	**57.61**
7.6	**57.76**	**57.91**	**58.06**	**58.22**	**58.37**	**58.52**	**58.68**	**58.83**	**58.83**	**59.14**
7.7	**59.29**	**59.44**	**59.60**	**59.75**	**59.91**	**60.06**	**60.22**	**60.37**	**60.37**	**6.68**
7.8	**60.84**	**61.00**	**61.15**	**61.31**	**61.47**	**61.62**	**61.78**	**61.94**	**61.94**	**62.25**
7.9	**62.41**	**62.57**	**62.73**	**62.88**	**63.04**	**63.20**	**63.36**	**63.52**	**63.52**	**63.84**
8.0	**64.00**	**64.16**	**64.32**	**64.48**	**64.64**	**64.80**	**64.96**	**65.12**	**65.29**	**65.45**
8.1	**65.61**	**65.77**	**65.93**	**66.10**	**66.26**	**66.42**	**66.59**	**66.75**	**66.91**	**67.08**
8.2	**67.24**	**67.40**	**67.57**	**67.73**	**67.90**	**68.06**	**68.23**	**68.39**	**68.56**	**68.72**
8.3	**68.29**	**69.06**	**69.22**	**69.39**	**69.56**	**69.72**	**69.89**	**70.06**	**70.22**	**70.39**
8.4	**70.56**	**70.73**	**70.90**	**71.06**	**71.23**	**71.40**	**71.57**	**71.74**	**71.91**	**72.01**
8.5	**72.25**	**72.42**	**72.59**	**72.76**	**72.93**	**73.10**	**73.27**	**73.44**	**73.62**	**73.79**
8.6	**73.96**	**74.13**	**74.30**	**74.48**	**74.65**	**74.82**	**75.00**	**75.17**	**75.34**	**75.52**
8.7	**75.69**	**75.86**	**76.04**	**76.21**	**76.39**	**76.56**	**76.74**	**76.91**	**76.09**	**77.26**
8.8	**77.44**	**77.62**	**77.79**	**77.97**	**78.15**	**78.32**	**78.50**	**78.68**	**78.85**	**79.03**
8.9	**79.21**	**79.39**	**79.57**	**79.74**	**79.92**	**80.10**	**80.28**	**80.46**	**80.46**	**80.82**
9.0	**81.00**	**81.18**	**84.36**	**81.54**	**81.72**	**81.90**	**82.08**	**82.26**	**82.45**	**82.63**
9.1	**82.81**	**82.99**	**83.17**	**83.36**	**83.54**	**83.72**	**83.91**	**84.09**	**86.12**	**84.46**
9.2	**84.64**	**84.82**	**85.01**	**85.19**	**85.38**	**85.56**	**85.75**	**84.93**	**87.96**	**86.30**
9.3	**86.49**	**86.68**	**86.86**	**87.05**	**87.24**	**87.42**	**87.61**	**87.80**	**87.96**	**88.17**
9.4	**88.36**	**88.55**	**88.74**	**88.92**	**89.11**	**89.30**	**89.49**	**89.68**	**89.87**	**90.06**
9.5	**90.25**	**90.44**	**90.63**	**90.82**	**91.01**	**91.20**	**91.39**	**91.58**	**91.78**	**91.97**
9.6	**92.16**	**92.35**	**92.54**	**92.74**	**92.93**	**93.12**	**93.32**	**93.51**	**93.70**	**93.90**
9.7	**94.09**	**94.28**	**94.67**	**94.67**	**94.87**	**95.06**	**95.26**	**95.45**	**95.65**	**95.84**
9.8	**96.04**	**96.24**	**96.43**	**96.63**	**96.83**	**97.02**	**97.22**	**97.42**	**97.61**	**97.81**
9.9	**98.01**	**98.21**	**98.41**	**98.60**	**98.80**	**99.00**	**99.20**	**99.40**	**99.60**	**99.80**

$(8.47)^2 = 71.74$ $(0.847)^2 = 0.7174$ $\sqrt{\mathbf{21.44}} = \mathbf{4.63}$ $\sqrt{\mathbf{0.2144}} = 0.463$

$(84.7)^2 = 7174$ $(8.472)^2 = 71.77$ $\sqrt{\mathbf{21.44}} = \mathbf{4.63}$

$Y = x^3$ $1.00 \leq x \leq 5.99$

X	0	1	2	3	4	5	6	7	8	9
1.0	1.000	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260	1.295
1.1	1.331	1.368	1.405	1.443	1.482	1.521	1.561	1.602	1.643	1.685
1.2	1.728	1772	1.816	1.861	1.907	1.953	2.000	2.048	2.097	2.147
1.3	2.197	2.248	2.300	2.353	2.406	2.460	2.515	2.571	2.628	2.686
1.4	2.744	2.803	2.863	2.924	2.986	3.049	3.112	3.177	3.242	3.308
1.5	3.375	3.443	3.512	3.582	3.652	3724	3.796	3.870	3.944	4.020
1.6	4.096	4.173	4.252	4.331	4.411	4.492	4.574	4.657	4.742	4.827
1.7	4.913	5.000	5.088	5.178	5.268	5.359	5.452	5.545	5.640	5.735
1.8	5.832	8.930	6.029	6.128	6.230	6.332	6.435	6.539	6.645	6.751
1.9	6.859	6.968	7.078	7.189	7.301	7.415	7.530	7.645	7.762	7.881
2.0	8.000	8.121	8.242	8.365	8.490	8.615	8.742	9.870	8.999	9.129
2.1	9.261	93.394	9.528	9.664	9.800	9.938	10.08	10.22	10.36	10.50
2.2	10.65	10.79	11.94	11.09	11.24	11.39	11.54	11.70	11.85	12.01
2.3	12.17	12.33	12.49	12.65	12.81	12.98	13.14	13.31	31.48	13.65
2.4	13.82	14.00	14.17	14.35	14.53	14.71	14.89	15.07	15.25	15.44
2.5	15.63	15.81	16.00	16.19	16.39	16.58	16.78	16.97	17.17	17.37
2.6	17.58	17.78	17.98	18.19	18.40	18.61	18.82	19.03	19.25	19.47
2.7	19.68	19.90	20.12	20.35	20.57	20.08	21.02	21.25	21.48	21.72
2.8	21.95	22.19	22.43	22.67	22.91	23.15	23.39	23.64	23.89	24.14
2.9	24.39	24.64	24.90	25.15	25.41	25.67	25.93	26.20	25.46	26.73
3.0	27.00	27.27	27.54	27.82	28.09	28.37	28.65	28.93	29.22	29.50
3.1	29.79	30.08	30.37	30.66	30.96	26	31.55	31.86	32.16	32.46
3.2	32.77	33.08	33.39	33.70	34.01	34.33	34.65	34.97	35.29	35.61
3.3	35.94	36.26	36.59	36.93	37.26	37.60	37.93	38.27	38.61	38.96
3.4	39.30	39.65	40.00	40.35	40.71	41.06	41.42	41.78	42.14	42.51
3.5	42.88	43.24	43.61	43.99	44.36	44.47	45.12	45.50	45.88	46.27
3.6	46.66	47.05	47.44	47.83	48.23	48.63	49.03	49.43	49.84	50.24
3.7	50.65	51.06	51.48	51.90	52.31	52.73	53.16	53.58	54.01	54.44
3.8	54.87	55.31	55.74	56.18	56.62	57.07	57.51	57.96	58.41	58.86
3.9	59.32	59.78	60.24	60.70	61.16	61.63	62.10	62.57	63.04	63.52
4.0	64.00	64.48	64.96	65.45	65.94	66.43	66.92	67.42	67.92	68.42
4.1	68.92	69.43	69.913	70.44	70.96	71.47	71.99	72.51	73.03	73.56
4.2	74.09	74.62	75.15	75.69	76.23	76.77	77.31	77.85	78.40	79.95
4.3	79.51	80.06	80.62	81.18	81.75	82.31	82.88	83.45	84.03	84.60
4.4	85.18	85.77	86.35	86.94	87.53	88.12	88.72	89.31	89.92	90.52
4.5	91.13	91.73	92.35	92.96	93.58	94.20	94.82	95.44	96.07	96.70
4.6	97.34	97.97	98.61	99.25	99.90	100.5	101.2	101.8	102.5	103.2
4.7	103.8	104.5	105.2	105.8	106.5	107.2	107.9	108.5	109.2	109.9
4.8	110.6	111.3	112.0	112.7	113.4	114.1	114.8	115.5	116.2	116.9
4.9	117.6	118.4	119.1	119.8	120.6	121.3	122.0	122.8	123.5	124.3
5.0	125.0	125.8	126.5	127.3	128.0	128.8	129.6	130.3	131.1	131.9
5.1	132.7	133.4	134.2	135.0	135.8	136.6	137.4	138.2	139.0	139.8
5.2	140.6	141.4	142.2	143.1	143.9	144.7	145.5	146.4	147.2	148.0
5.3	148.9	149.7	150.6	151.4	152.3	153.1	154.0	154.5	155.7	156.6
5.4	157.5	158.3	159.2	160.1	161.0	161.9	162.8	163.7	164.6	165.5
5.5	166.4	167.3	168.2	169.1	170.0	171.0	171.9	172.8	173.7	174.7
5.6	175.6	176.6	177.5	178.5	179.4	180.4	181.3	182.3	183.3	184.2
5.7	185.2	186.2	187.1	188.1	189.1	190.1	191.1	192.1	193.1	194.1
5.8	195.1	196.1	197.1	198.2	199.2	200.2	201.2	202.3	203.3	204.3
5.9	205.4	206.4	207.5	208.5	209.6	210.6	211.7	212.8	213.8	214.9

If you move the comm. In x one digit to the right (left), then the comma in x^3 must be moved three digits to the right (left)

$Y = x^3$ $6.00 \leq x \leq 9.99$

X	0	1	2	3	4	5	6	7	8	9
6.0	216.0	217.1	218.2	219.3	220.3	221.4	211.7	223.6	224.8	225.9
6.1	227.0	228.1	229.2	230.0	231.5	232.6	222.5	234.9	236.0	237.2
6.2	238.3	239.5	240.6	241.8	243.0	244.1	233.7	246.5	247.7	248.9
6.3	250.0	251.2	252.4	253.6	254.8	256.0	245.3	258.5	259.7	260.9
6.4	262.1	263.4	264.6	265.8	267.1	268.3	257.3	270.8	272.1	273.4
6.5	274.6	275.9	277.2	278.4	279.7	281.0	269.6	283.6	284.9	286.2
6.6	287.5	288.8	290.1	291.4	292.8	294.1	282.3	296.7	298.1	299.4
6.7	300.8	302.1	303.5	304.8	306.2	307.5	295.4	310.3	311.7	313.0
6.8	314.4	315.8	317.2	318.6	320.0	321.4	308.9	324.2	325.7	327.1
6.9	328.5	329.9	331.4	332.8	334.3	335.7	322.8	324.2	340.1	341.5
7.0	343.0	344.5	345.9	347.4	348.9	350.4	337..2	338.6	354.9	356.4
7.1	357.9	359.4	360.9	362.5	364.0	365.5	351.9	368.6	370.1	371.7
7.2	373.2	374.8	376.4	377.9	379.5	381.1	367.1	384.2	385.8	387.4
7.3	389.0	390.6	392.2	393.8	395.4	397.1	382.7	400.3	401.9	403.6
7.4	405.2	406.9	408.5	410.2	411.5	413.5	398.7	416.8	418.5	420.2
7.5	421.9	423.6	425.3	427.0	428.7	430.4	415.2	433.8	435.5	437.2
7.6	439.0	440.7	442.5	444.2	445.9	447.7	432.1	451.2	453.0	454.8
7.7	456.5	458.3	460.1	461.9	463.7	465.5	449.5	469.1	470.9	472.7
7.8	474.6	476.4	478.2	480.0	481.9	483.7	467.3	487.4	489.3	491.2
7.9	493.0	494.9	496.8	498.7	500.6	502.5	485.6	506.3	508.2	510.1
8.0	512.0	513.9	515.8	514.8	519.7	521.7	504.4	525.6	527.5	529.5
8.1	531.4	533.4	535.4	537.4	539.4	541.3	523.6	545.3	547.3	549.4
8.2	551.4	533.4	555.4	557.4	559.5	561.5	543.3	565.6	567.7	569.7
8.3	571.8	573.9	575.9	578.0	580.1	582.2	563.6	586.4	588.5	590.6
8.4	592.7	594.8	596.9	599.1	601.2	603.4	584.3	607.6	609.8	612.0
8.5	614.1	616.3	618.5	620.7	622.8	625.0	605.5	629.4	631.6	633.8
8.6	636.1	638.3	640.5	642.7	645.0	647.2	627.2	651.7	654.0	656.2
8.7	658.5	660.8	663.1	665.3	667.6	669.9	649.5	674.5	676.8	679.2
8.8	681.5	683.8	686.1	688.5	690.8	693.2	672.2	697.9	700.2	702.6
8.9	705.0	707.3	709.7	712.1	714.5	716.9	695.5	721.7	724.2	726.6
9.0	729.0	731.4	733.9	736.3	738.8	741.2	719.3	746.1	748.6	751.1
9.1	753.6	756.1	758.6	761.0	763.6	766.1	743.7	771.1	773.6	776.2
9.2	778.7	781.2	783.8	786.3	788.9	791.5	768.6	796.6	799.2	801.8
9.3	804.4	807.0	809.6	812.2	814.8	817.4	794.0	822.7	825.3	827.9
9.4	830.6	833.2	835.9	838.6	814.2	843.9	820.0	849.3	852.0	854.7
9.5	857.4	860.1	862.8	865.5	868.3	871.0	846.6	876.5	879.2	882.0
9.6	884.7	887.5	8890.3	893.1	895.8	898.6	873.7	904.2	907.0	909.9
9.7	912.7	915.5	918.3	921.2	924.0	926.9	901.4	932.6	935.4	938.3
9.8	941.2	944.1	947.0	449.0	952.8	955.7	929.7	961.5	964.4	967.4
9.9	970.3	973.2	976.2	979.	982.1	985.1	958.6	991.0	994.0	997.0

$(8.47)^3 = 607.6$ $(0.847)^3 = 0.607$ $\sqrt[3]{123.5} = 4.98$ $\sqrt[3]{0.1235} = 0.498$

$(84.7)^3 = 607600$ $(8.472)^3 = 608.0$ $\sqrt[3]{123500} = 49.8$